Physica E 41 (2008) 1–5

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Fractional quantum hall effect and electron correlations in partially filled first excited Landau level

George E. Simion *, John J. Quinn

Department of Physics and Astronomy, University of Tennessee, Knoxville, TN 37996, USA

ARTICLE INFO

Article history:
Received 6 April 2008
Accepted 8 April 2008
Available online 29 April 2008

PACS:
71.10.Pm
73.43.Cd

Keywords:
Fractional quantum Hall effect
Strongly interacting electrons
Landau level

ABSTRACT

We present a quantitative study of most prominent incompressible quantum Hall states in the partially filled first excited Landau level (LL1) which have been recently studied experimentally by Choi et al. The pseudopotential describing the electron–electron interaction in LL1 is harmonic at short range. It produces a series of incompressible states which is different from its LL0 counterpart. The numerical data indicate that the most prominent states $\nu=\frac{5}{2}, \frac{7}{3}$ and $\frac{8}{3}$ are not produced by Laughlin correlated electrons, but result from a tendency of electrons to form pairs or larger clusters which eventually become Laughlin correlated. States with smaller gaps at filling factors $\frac{14}{5}, \frac{16}{7}, \frac{11}{5}, \frac{19}{7}$ are Laughlin correlated electron or hole states and fit Jain's sequence of filled CF^4 levels.

1. Introduction

The possibility of using non-Abelian quasiparticle excitations in quantum computing has led to a revival of interest in the fractional quantum Hall (FQH) states of the first excited Landau level (LL1) [1–5]. The FQH states of LL1 are quite different from those of LL0, and not nearly as well understood [6–12]. Before making use of the properties of non-Abelian quasiparticles (QPs) of LL1 (of a very special model three-body interaction) for practical purposes, it seems worthwhile to study the incompressible quantum liquid (IQL) states and their QPs in LL1 using realistic Coulomb interactions. A recent careful study of the temperature dependence of the minimum longitudinal conductivity [13] noted that the most robust FQH states in LL1 occur at $\nu_1=\nu-2=\frac{1}{2}, \frac{1}{3}, \frac{1}{5}$ and $\frac{2}{7}$ and their electron–hole (e–h) conjugates. It was emphasized that this was in sharp contrast to LL0, where $\nu=\frac{1}{3}, \frac{2}{5}, \frac{3}{7}$ and their e–h conjugate were the most prominent incompressible states, but no discussion was given of why this difference occurred. In this paper we present an explanation of why correlations in LL0 and LL1 are different and result in different sets of prominent FQH states. We support our explanation with numerical results for energy spectra and for the probability $P(\mathcal{R})$ that the ground state contains pairs with relative angular momentum $\mathcal{R}=1,3,5,\ldots$. We restrict our consideration to states that are fully spin polarized for the sake of simplicity, and because they appear to yield agreement with experimental observations.

2. Pseudopotentials and Laughlin correlations

In this paper we use Haldane's spherical geometry [14], in which there are no boundary conditions to be imposed and the planar translational symmetry is replaced by the spherical one. The perpendicular magnetic field is produced by a monopole with the strength $2Q$ and the Landau levels n are replaced by angular momentum shells $\ell=Q+n$. The Coulomb interaction is described using the pseudopotentials $V(\mathcal{R})$, where $\mathcal{R}$ is the relative angular momentum $\mathcal{R}=2\ell-L'$, L' being the pair angular momentum. It is well known that Laughlin correlations (the avoidance of pair states with small values of $\mathcal{R}$) occur only when the pseudopotential $V_n(\mathcal{R})$ describing the interaction energy of a electron pair with angular momentum $L'=2\ell-\mathcal{R}$ in LLn is "superharmonic", i.e. rises with increasing L' faster than $L'(L'+1)$ as the avoided value of L' is approached [15–17]. In LL0 the pseudopotential is superharmonic for all values of $\mathcal{R}$, but in LL1 it is superharmonic only for $R\geqslant 3$. Finite well-width effects can make $V_1(R)$ weakly superharmonic at $\mathcal{R}=1$ [18]. However, for well width of moderate size, we have found that $V_1(\mathcal{R})$ does not change enough to cause robust Laughlin correlations. We will ignore finite width effect in this paper.

When $\frac{1}{2}\geqslant\nu\geqslant\frac{1}{3}$, the lowest energy states in LL0 contain Laughlin correlated electrons (LCEs) which avoid pairs at $\mathcal{R}=1$. A Laughlin–Jain [19,20] sequence of integrally filled LCE levels occurs at

* Corresponding author. Tel.:+1 865 9740771.
E-mail address: gsimion@utk.edu (G.E. Simion).

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doi:10.1016/j.physe.2008.04.004

$\nu = n(2n \pm 1)^{-1}$, when the composite Fermion (CF) angular momentum $\ell^* = |\ell - (N-1)|$ satisfies $2\ell^* = N/n - n$. For $\frac{1}{3} > \nu \geqslant \frac{1}{5}$, the LCEs avoid pair states with $\mathscr{R} = 1$ and 3, giving $\ell^* = |\ell - 2(N-1)|$ and $\nu = n(4n \pm 1)^{-1}$ FQH states.

3. Correlations in the first excited Landau level

$V_1(\mathscr{R})$ is not superharmonic [10,11] at $\mathscr{R} = 1$, so LCEs are not expected for $\frac{1}{2} \geqslant \nu \geqslant \frac{1}{3}$. Instead, the electrons tend to form pairs with

$$\ell_P = 2\ell - 1. \quad (1)$$

The pairs of electrons can be treated as Bosons or Fermions since in 2D systems a Chern–Simons transformation can change Bosons to Fermions [21]. To avoid violating the exclusion principle, we cannot allow Fermion pairs (FPs) to be too close to one another. We do this by restricting the angular momentum of two pairs to values less than or equal to [22–25]

$$2\ell_{FP} = 2\ell_P - 3(N_P - 1), \quad (2)$$

where the number of pairs is

$$N_P = \frac{N}{2}. \quad (3)$$

From this it is apparent that the FP filling factor satisfies the relation $\nu_{FP}^{-1} = 4\nu_1^{-1} - 3$. The factor of 4 is a reflection of N_P being half of N and the LL degeneracy g_P of the pairs being twice g for electrons. Correlations are introduced through a standard CF transformation applied to FPs:

$$2\ell_{FP}^* = 2\ell_{FP} - 2p(N_P - 1). \quad (4)$$

For $(2\ell, N) = (25, 14)$ and $2p = 4$ this gives $\ell_P = 24$, $N_P = 7$, $2\ell_{FP} = 30$ and $2\ell_{FP}^* = 6$. The $N_P = 7$ FPs fill the $\ell_{FP}^* = 3$ shell giving an $L = 0$ IQL ground state with FP filling factor $\nu_{FP} = (2p+1)^{-1} = \frac{1}{5}$ and $\nu_1 = \frac{1}{2}$. The numerical spectrum is shown in Fig. 1a; it has a well-defined gap separating the ground state from the excited states. $P(\mathscr{R})$ vs. $\mathscr{R}$ is obtained from the eigenfunctions and contains the same information for a spherical surface as the pair distribution function on a plane. The maximum at $\mathscr{R} = 1$ and minimum at $\mathscr{R} = 3$ is incontrovertible evidence that the electrons are not Laughlin correlated. Our $L = 0$ ground state of Laughlin correlated pairs is definitely different form the Moore–Read Pfaffian state [6], the exact eigenstate of a special three particle repulsive interaction [7,11]. The square of the overlap of these two ground state wavefunctions is only 0.48; the overlap of the excited states for the two models is much smaller.

In Fig. 2 we present the two energy spectra for $N = 14$ and $2\ell = 24$ (a) and 26 (b). Increasing or decreasing 2ℓ (from the values that give an IQL ground state) by one unit clearly produces a pair of elementary excitations and a band of low lying states. We can understand these spectra using our set of equations (1)–(4) for paired states with $(2\ell, N) = (24, 14)$ and (26,14). The former case contains two FP quasiparticles each with $\ell_{FPQP} = 3$, and the latter two FP quasiholes with $\ell_{FPQH} = 4$, giving the lowest energy bands marked by circled dots in Fig. 2. The occurrence of FQH states at $2\ell = 2N - 3$ and at the electron–hole conjugate $2\ell = 2N + 1$ (obtained by replacing N with $2\ell + 1 - N$) agrees with numerical calculations. The gap appears to approach a finite limit for large N when plotted as a function of N^{-1}, but our calculations are limited $8 \leqslant N \leqslant 16$. A meaningful comparison with experimental data would require taking into account the finite well width, the effect of disorder and especially LL mixing as suggested by very recent experimental data [26]. The low lying states for values of 2ℓ close to $2N - 3$ seem to be well described in terms of Laughlin correlated pairs of electrons with the pairs treated as Fermions.

For $\nu_1 = \frac{1}{3}$ in LL1, we performed numerical calculations for many different values of 2ℓ for each value of N [10,11]. $L = 0$ ground states were found for $6 \leqslant N \leqslant 12$ at $2\ell = 3N - 7$, quite a different value than for the $L = 0$ incompressible state in LL0, which occur at $2\ell = 3N - 3$. Although pairs are expected to form, complete pairing for even values of N would give an incompressible ground state at $2\ell = 3N - 5$, not $3N - 7$. We do not fully understand the correlations at $\nu_1 = \frac{1}{3}$; larger cluster than pairs could form or we could have a plasma with clusters of several different sizes. Such systems might be treated by a generalized CF picture [27], but we do not consider a plasma with two or more different cluster sizes in the present paper. In Fig. 3 we show an example of a spectrum, $N = 11$ and $2\ell = 26$, and a plot of $P(\mathscr{R})$ vs. $\mathscr{R}$ for the $L = 0$ ground state. It is clear that the electrons are not Laughlin correlated since $P(R)$ is a minimum for $\mathscr{R} = 3$, not $\mathscr{R} = 1$. Unfortunately, the gap size is not a smooth function of N^{-1}, so our limited range of N values does not allow extrapolation to the macroscopic limit with any degree of confidence. We are certain, however, that pairs or larger clusters will form instead of LCEs.

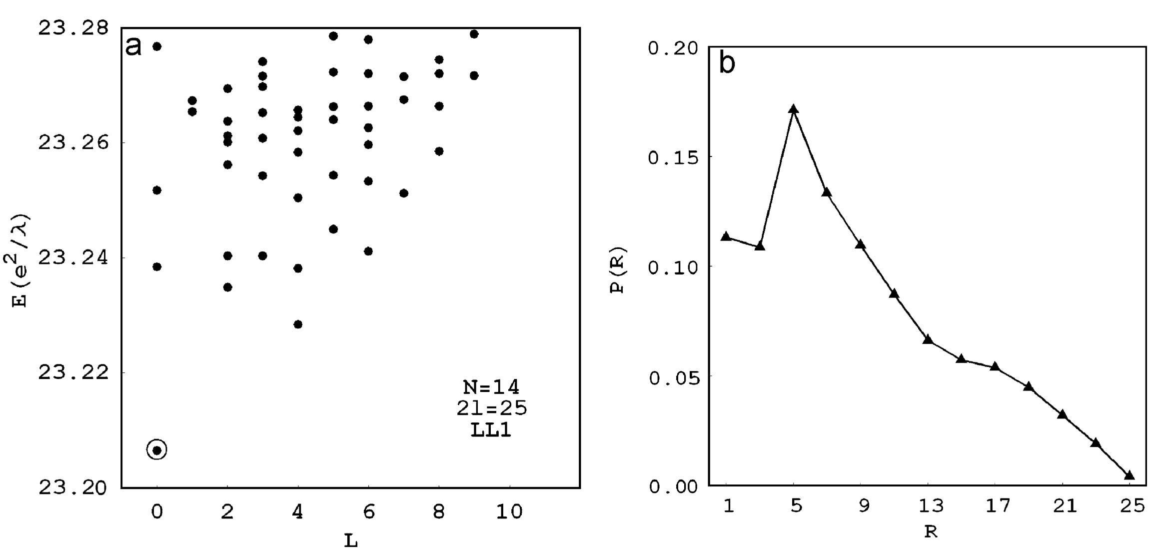

Fig. 1. (a) The low energy spectrum of $N = 14$ electrons in LL1 at $2\ell = 25$. The $L = 0$ IQL state, marked by a circle, is separated from higher state by a gap. (b) The probability $P(\mathscr{R})$ of pair for $L = 0$ ground state with relative angular momentum $\mathscr{R}$ as a function of $\mathscr{R}$. $P(\mathscr{R})$ is not a minimum at $\mathscr{R} = 1$, and not a maximum at $\mathscr{R} = 3$ as in a Laughlin state.