Available online at www.sciencedirect.com
ScienceDirect
Physica E 40 (2007) 76–83
PHYSICA E
www.elsevier.com/locate/physe

Spin dephasing due to a random Berry phase

Pablo San-Jose[a], Gerd Schön[a], Alexander Shnirman[a], Gergely Zarand[a,b,*]

[a]*Institut für Theoretische Festkörperphysik and DFG-Center for Functional Nanostructures (CFN), Universität Karlsruhe, D-76128 Karlsruhe, Germany*
[b]*Institute of Physics, Technical University Budapest, Budapest H-1521, Hungary*

Available online 16 June 2007

Abstract

We investigate relaxation and dephasing of an electron spin confined in a semiconductor quantum dot and subject to spin–orbit coupling. Even in vanishing magnetic field, $\boldsymbol{B} = 0$, slow noise coupling to the electron's orbital degree of freedom leads to dephasing of the spin due to a random, in general non-Abelian Berry phase acquired by the spin. For illustration we first present a simple quasiclassical description, then consider a model with two orbital states only, and finally present a perturbative quantum treatment appropriate for an electron in a realistic (roughly parabolic, not too strongly confining) quantum dot. We further compare the effect of different sources of noise. While at large magnetic fields phonons dominate the relaxation processes, at low fields electron–hole excitations and possibly $1/f$ noise may dominate.

PACS: 72.25.Rb; 70.71.Ej; 03.65.Vf

Keywords: Spin relaxation; Spin-orbit; Geometrical phases

1. Introduction

The demonstrations of coherent single-electron spin control and measurement [1–3] in semiconductor quantum dots have opened exciting perspectives for solid-state quantum information processing with spin qubits [4]. More recent work [5–8] has revealed further potential of spin coherence, which greatly extends the possibilities of next-generation spintronic devices. The key behind these emerging technologies is the long spin coherence time in semiconductor materials. Spins, unlike orbital electron degrees of freedom, do not couple directly to the various sources of electric noise present in typical solid-state environments.

Most of the traditional techniques for addressing and manipulating spins in semiconductors have revolved around some form of electron spin resonance (ESR), be it through external magnetic fields [3] or effective internal ac fields based on the spin–orbit interaction. Indeed, spin–orbit interaction has been proposed theoretically as a way of coherently controlling the spin of confined electrons purely by electrical means [9–13], and important experimental progress has been made in this direction [14,15]. By the same token, it has long been understood [16,17] that spin–orbit interaction is one of the main mechanisms by which electron spins decay and lose coherence in semiconductor heterostructures [18–21].

As we will discuss in this paper, in the particular case of an electron confined in a quantum dot, a time-dependent (fluctuating or controlled) electric field introduces via the spin–orbit coupling a non-Abelian geometric phase (a generalization of Berry phases) into the spin evolution. This connection between spin–orbit interaction and geometric phases has been noted previously in the context of perturbative analysis of the spin decay of trapped electrons [22]. A similar connection had been discussed for free electrons in the presence of disorder scattering [23].

The geometric character of spin evolution under electric fields has striking consequences both for spin–orbit mediated spin relaxation and decoherence as well as for coherent spin manipulation strategies. Geometric spin

*Corresponding author. Institute of Physics, Technical University Budapest, Budapest H-1521, Hungary. Tel.: +36 14634107; fax: +36 14633567.
E-mail address: zarand@newton.phy.bme.hu (G. Zarand).

1386-9477/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.physe.2007.05.027

evolution under controlled gating is potentially robust, since it is not affected by gate timing errors and certain control voltage inaccuracies. In the case of spin decay, the non-Abelian character of the spin precession under a noisy electric environment results in a saturation of spin relaxation rates at low magnetic fields [16] through a fourth order (in the spin–bath coupling) process previously overlooked in the literature [19,24,25]. This spin decay mechanism, which can be called *geometric dephasing*, requires two independent noise sources coupled to two non-commuting components of the electron spin, whereby the non-Abelian properties of the SU(2) group become relevant.[1] To second order in the spin–bath coupling we also note that a different source of fluctuations other that piezoelectric phonons, namely electron–hole excitations in the metallic environment (ohmic fluctuations), dominate the spin relaxation at low magnetic fields. The reason is the higher density of ohmic fluctuations at low energies as compared to phonons.

The present paper is organized as follows. In Section 2 we will present qualitatively the main concepts and consequences of the geometric character of the electrically induced spin precession in leading order in the ratio between dot size to spin–orbit length, $x_0/l_{\rm so}$. In Section 3 we consider a model system based on only two orbital states of an electron with spin. This model helps to understand the geometrical evolution of the spin. In Section 4 we will perturbatively derive the effective Hamiltonian for an electron in a quantum dot under a fluctuating electric field taking into account all orbital states. This will allow us to analyze the spin relaxation and dephasing under realistic conditions.

2. Geometric spin precession of a strongly confined electron

Electric fields applied to a quantum dot structure induce displacements (and possibly deformations) in the confining potential. In the presence of spin–orbit interaction this will lead to a peculiar geometric evolution of the spin state of the confined electron with important consequences for the relaxation and manipulation of the spin. We will approach the problem by first considering the spin precession due to a geometric phase acquired by the spin of a strongly confined electron, when it is adiabatically transported along a *given trajectory* in a 2DEG in the presence of spin–orbit coupling.

In semiconductor 2D heterostructures the spin–orbit coupling takes the form ($\hbar = 1$ throughout this work)

$$\mathscr{H}_{\rm so} = \alpha(\hat{p}_y\hat{\sigma}_x - \hat{p}_x\hat{\sigma}_y) + \beta(\hat{p}_y\hat{\sigma}_y - \hat{p}_x\hat{\sigma}_x) = \frac{1}{m}\,\hat{\boldsymbol{p}}\boldsymbol{\lambda}_{\rm so}^{-1}\hat{\boldsymbol{\sigma}}. \tag{1}$$

[1] A different phenomenon, also called geometric dephasing, was discussed in Ref. [26]. There geometric manipulations of spins in finite magnetic fields and the presence of dissipation were considered and path-dependent (geometric) contributions to dephasing were found.

Here $\hat{\boldsymbol{\sigma}}/2$ and $\hat{\boldsymbol{p}}$ are the spin and momentum operators, while α and β are the Rashba and linear Dresselhaus couplings, which can be lumped into the spin–orbit tensor

$$\boldsymbol{\lambda}_{\rm so}^{-1} \equiv m\begin{pmatrix} -\beta & -\alpha \\ \alpha & \beta \end{pmatrix}. \tag{2}$$

It sets the scale for the spin–orbit length $l_{\rm so} \equiv \sqrt{|\det \boldsymbol{\lambda}_{\rm so}|} = (m\sqrt{|\alpha^2-\beta^2|})^{-1}$. The effective strength of the spin–orbit effects in a quantum dot of size x_0 is in general proportional to some power of the ratio $x_0/l_{\rm so}$. In typical GaAs/AlGaAs semiconductor heterostructures $l_{\rm so}\sim 1-5\,\mu$m, while $x_0\sim$30–100 nm so that this ratio is usually quite small, of the order of 0.02. Other materials, such as InAs, have a much stronger spin–orbit length, in the $l_{\rm so}\sim$100 nm range.

By classical intuition we can anticipate the main effect. We consider an electron in a very strong confinement, forced to move along a path $\mathscr{C}$ with trajectory $\boldsymbol{R}_{\mathscr{C}}(t)$. Eq. (1) suggests that the spin–orbit coupling makes the spin precess under an effective magnetic field $\boldsymbol{B}_{\rm so} = (\frac{1}{m})\hat{\boldsymbol{p}}\boldsymbol{\lambda}_{\rm so}^{-1}$, which couples to the spin similar to a Zeeman term except that the field depends on the electron's momentum. It raises the question as to what "value" one should use for operator $\hat{\boldsymbol{p}}$. For a strongly confining potential it turns out that we can simply substitute $\hat{\boldsymbol{p}} \to m\dot{\boldsymbol{R}}_{\mathscr{C}}$. Hence,

$$\boldsymbol{B}_{\rm so} = \dot{\boldsymbol{R}}_{\mathscr{C}}\boldsymbol{\lambda}_{\rm so}^{-1}. \tag{3}$$

From this we derive a spin precession governed by the following SU(2) operator:

$$U_{\rm ad}(t) = T\exp\left(-{\rm i}\int_0^t {\rm d}t\,\boldsymbol{B}_{\rm so}\cdot\hat{\boldsymbol{\sigma}}\right) = P\exp\left(-{\rm i}\int_{\mathscr{C}} {\rm d}\boldsymbol{R}_{\mathscr{C}}\,\boldsymbol{\lambda}_{\rm so}^{-1}\hat{\boldsymbol{\sigma}}\right). \tag{4}$$

Here T and P stand for time- and path-ordering operators, respectively. The label "adiabatic" in $U_{\rm ad}$ refers to the constraint of slow paths, $|\dot{\boldsymbol{R}}_{\mathscr{C}}| \ll x_0\omega_0$, typically assumed in most works on Berry phases [27]. As is apparent from Eq. (4), due the peculiar dependence of $\boldsymbol{B}_{\rm so}$ on the velocity $\dot{\boldsymbol{R}}_{\mathscr{C}}$, the total "geometric spin precession" for propagation along a given path $\mathscr{C}$ depends only on the geometry of $\mathscr{C}$ itself, not on the time dependence of $\boldsymbol{R}_{\mathscr{C}}$.

Another line of arguments leading to this result was pointed out in Ref. [28]. It is based on the observation that $H_{\rm so}$ can be diagonalized to first order in $x_0/l_{\rm so}$ by a canonical transformation $\exp(-{\rm i}\hat{\boldsymbol{r}}\boldsymbol{\lambda}_{\rm so}^{-1}\hat{\boldsymbol{\sigma}})$, which in turn implies that in a small dot the effect of spin–orbit coupling moving along a given path can be gauged away by a path-dependent gauge transformation $U_{\rm ad}$ that rotates the spin just as in Eq. (4).

The evolution operator $U_{\rm ad}$ is a group element in SU(2). However, it can also be mapped onto a SO(3) rotation of a 3D solid, since both groups are isomorphic up to a sign. The natural question arises, what is the 3D rotation corresponding to $U_{\rm ad}$ for a given path? Is there an intuitive