Available online at www.sciencedirect.com
ScienceDirect
Physica E 40 (2007) 163–168
PHYSICA E
www.elsevier.com/locate/physe

Ubiquitous presence of odd-frequency pairing state in superconducting junctions

A.A. Golubov[a,*], Y. Tanaka[b,c], S. Kashiwaya[d], M. Ueda[e]

[a]Faculty of Science and Technology, University of Twente, The Netherlands
[b]Department of Applied Physics, Nagoya University, Nagoya 464-8603, Japan
[c]CREST Japan Science and Technology Cooperation (JST), 464-8603, Japan
[d]National Institute of Advanced Industrial Science and Technology (AIST), Tsukuba 305-8568, Japan
[e]Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan

Available online 21 May 2007

Abstract

We predict generation of the odd-frequency pairing state near interfaces in conventional even-frequency superconductors. Using the quasiclassical Green's function formalism, we show that the pair amplitude can be decomposed into even and odd-frequency components. We demonstrate that, quite generally, the spin-singlet even-parity (spin-triplet odd-parity) pair potential in a superconductor induces the odd-frequency pairing component with spin-singlet odd-parity (spin-triplet even-parity) near interfaces. The magnitude of the induced odd-frequency component is enhanced in the presence of the midgap Andreev resonant state due to the sign change of the anisotropic pair potential at the interface. The Josephson effect should therefore occur between odd- and even-frequency superconductors, contrary to the standard wisdom. A method to probe the odd-frequency superconductors is proposed.

PACS: 74.45.+c; 74.50.+r; 74.20.Rp

Keywords: Superconductivity; Unconventional pairing; Proximity effect; Andreev bound states

Exploring novel pairing states in the field of unconventional superconductivity became a challenging issue in solid state physics [1]. The Pauli principle imposes the constraint on a superconducting state requiring the condensate wave function to be odd with respect to the permutation of electrons. Therefore, spin-singlet even-parity and spin-triplet odd-parity pairing states are realized when the pair amplitude is an even function of energy (or Matsubara frequency). In contrast to that, the realization of spin-singlet odd-parity and spin-triplet even-parity pairing states are known to be possible when the pair amplitude is an odd function of the frequency.

The possibility of the odd-frequency pairing state in a uniform system was firstly proposed by Berezinskii [2] who discussed the hypothetical odd-frequency spin-triplet pairing in the context of pairing mechanism of ^{3}He. This idea was recently extended to various mechanisms of superconductivity involving strong correlations [3,4]. On the other hand, the realization of the odd-frequency pairing state without finite pair potential was proposed in Ref. [5] in ferromagnet/superconductor heterostructures with inhomogeneous magnetization, and several related works have been presented up to now [6]. Recent experiments [7] provided the evidence of the presence of such anomalous pairing states. However, the odd-frequency pairing has been discussed in these limited situations so far.

The fundamental question arises whether the generation of similar odd-frequency pairing state is possible in the superconducting hybrid structures without magnetic ordering. The classical example is a normal metal/superconductor (N/S) junction with a spin-singlet d-wave or a spin-triplet p-wave superconductor having the conventional even-frequency pairing in the bulk. There are two distinct physical effects that may cause anomalies in such system. First, the magnitude of the pair potential is suppressed near

*Corresponding author. Tel.: +31 53 489 3122; fax: +31 53 489 1099.
E-mail address: a.golubov@tnw.utwente.nl (A.A. Golubov).

1386-9477/$ - see front matter
doi:10.1016/j.physe.2007.05.004

interfaces in anisotropic superconductors due to the scattering of quasiparticles at the interface [8]. There is a chance that the even-parity and the odd-parity Cooper pairs can coexist once the translational symmetry is broken and the pair amplitude has spatial variation. Second, the midgap Andreev resonant states (MARS) are formed near interfaces in spin-singlet d-wave or spin-triplet p-wave superconductors depending on the orientation of the junction [22]. The MARS are related to the topological nature of the orbital part of the pair potential since the internal phase of the pair potential is the essential ingredient for the MARS formation. The appearance of unusual charge transport in the presence of MARS [9] suggests the presence of underlying anomalous pairing states.

In the present paper, we study the pair amplitude in the generic case of an N/S interface using the quasiclassical Green's function theory where the spatial dependence of the pair potential is determined self-consistently. The superconductor is assumed to have the conventional even-frequency pairing state in the bulk, being in the spin-singlet even-parity state (s-wave or d-wave symmetry) or in the spin-triplet odd-parity state (p-wave symmetry). We show that, quite generally, the odd-frequency component is induced near the interface due to the spatial variation of the pair potential [10]. This result indicates that *the odd-frequency pairing state ubiquitously appears in non-uniform superconducting system*. Moreover, it is revealed that when the superconductor is in the even-parity (odd-parity) state, the resulting odd-frequency component is odd-parity (even-parity) in order to conserve the spin component. In the absence of the MARS, like in the case of s-wave or $d_{x^2-y^2}$-wave junctions, the magnitude of the odd-frequency component of the pair amplitude is suppressed with the decrease of the transmission coefficient across the interface. On the other hand, in the presence of the MARS, like in p_x-wave or d_{xy}-wave junctions, the magnitude of the odd-frequency component is enhanced with the decrease of the transmission coefficient. In the low transparent limit, only the odd-frequency component exists at the interface. These trends suggest the close correlation between the odd-frequency pairing state and the MARS.

In the following, we consider an N/S junction as the simplest example of non-uniform superconducting system without impurity scattering. Both the spin-triplet odd-parity and the spin-singlet even-parity states are considered in the superconductor. As regards the spin-triplet superconductor, we choose $S_z = 0$ for simplicity. We assume a thin insulating barrier located at the N/S interface ($x = 0$) with N ($x<0$) and S ($x>0$) modeled by a delta function $H\delta(x)$, where H is the magnitude of the strength of the delta function potential. The reflection coefficient of the junction for the quasiparticle for the injection angle θ is given by $R = Z^2/(Z^2 + 4\cos^2\theta)$ with $Z = 2H/v_F$, where θ ($-\pi/2<\theta<\pi/2$) is measured from the normal to the interface and v_F is the Fermi velocity. The quasiclassical Green's function in a superconductor is parameterized as

$$\hat{g}_\pm = f_{1\pm}\hat{\tau}_1 + f_{2\pm}\hat{\tau}_2 + g_\pm\hat{\tau}_3, \quad \hat{g}_\pm^2 = \hat{1} \tag{1}$$

with Pauli matrices $\hat{\tau}_i$ ($i = 1-3$) and unit matrix $\hat{1}$. Here, the index $+$ ($-$) denotes the left (right) going quasiparticles [11]. It is possible to express the above Green's function as $f_{1\pm} = \pm i(F_\pm + D_\pm)/(1 - D_\pm F_\pm)$, $f_{2\pm} = -(F_\pm - D_\pm)/(1 - D_\pm F_\pm)$, and $g_\pm = (1 + D_\pm F_\pm)/(1 - D_\pm F_\pm)$, where $D_\pm$ and $F_\pm$ satisfy the Eilenberger equations in the Riccatti parameterization [12]:

$$v_{Fx}\partial_x D_\pm = -\bar{\Delta}_\pm(x)(1 - D_\pm^2) + 2\omega_n D_\pm, \tag{2}$$

$$v_{Fx}\partial_x F_\pm = -\bar{\Delta}_\pm(x)(1 - F_\pm^2) - 2\omega_n F_\pm, \tag{3}$$

where v_{Fx} is the x component of the Fermi velocity, $\omega_n = 2\pi T(n + \frac{1}{2})$ is the Matsubara frequency, with temperature T. $\bar{\Delta}_+(x)$ ($\bar{\Delta}_-(x)$) is the effective pair potential for left going (right going) quasiparticles. Since the interface is flat, $F_\pm = -RD_\mp$ holds at $x = 0$ [12]. Here we consider the situation without mixing of different symmetry channels for the pair potential. Then $\bar{\Delta}_\pm(x)$ is expressed by $\bar{\Delta}_\pm(x) = \Delta(x)\Phi_\pm(\theta)\Theta(x)$ with the form factor $\Phi_\pm(\theta)$ given by 1, $\cos 2\theta$, $\pm\sin 2\theta$, $\pm\cos\theta$, and $\sin\theta$ for s, $d_{x^2-y^2}$, d_{xy}, p_x and p_y-wave superconductors, respectively. $\Delta(x)$ is determined by the self-consistency equation

$$\Delta(x) = \frac{2T}{\log(T/T_C) + \sum_{n\geqslant 1}(1/(n-1/2))}\sum_{n\geqslant 0}\int_{-\pi/2}^{\pi/2} \times d\theta G(\theta) f_{2+} \tag{4}$$

with $G(\theta) = 1$ for s-wave case and $G(\theta) = 2\Phi(\theta)$ for other cases, respectively [13]. T_C is the transition temperature of the superconductor. The condition in the bulk is $\Delta(\infty) = \Delta$. Since the pair potential $\bar{\Delta}(x)$ is a real number, the resulting $f_{1\pm}$ is an imaginary and $f_{2\pm}$ is a real number.

In the following, we explicitly write $f_{1\pm} = f_{1\pm}(\omega_n, \theta)$, $f_{2\pm} = f_{2\pm}(\omega_n, \theta)$, $F_\pm = F_\pm(\omega_n, \theta)$ and $D_\pm = D_\pm(\omega_n, \theta)$. For $x = \infty$, we obtain $f_{1\pm}(\omega_n, \theta) = 0$ and $f_{2\pm}(\omega_n, \theta) = \Delta\Phi_\pm(\theta)\Big/\sqrt{\omega_n^2 + \bar{\Delta}^2\Phi_\pm^2(\theta_\pm)}$. Note that $f_{1\pm}(\omega_n, \theta)$ becomes finite due to the spatial variation of the pair potential and does not exist as the bulk. From Eqs. (2) and (3), we can show $D_\pm(-\omega_n, \theta) = 1/D_\pm(\omega_n, \theta)$ and $F_\pm(-\omega_n, \theta) = 1/F_\pm(\omega_n, \theta)$. After a simple manipulation, we obtain $f_{1\pm}(\omega_n, \theta) = -f_{1\pm}(-\omega_n, \theta)$ and $f_{2\pm}(\omega_n, \theta) = f_{2\pm}(-\omega_n, \theta)$ for any x. It is remarkable that functions $f_{1\pm}(\omega_n, \theta)$ and $f_{2\pm}(\omega_n, \theta)$ correspond to odd-frequency and even-frequency components of the pair amplitude, respectively [5].

Next, we shall discuss the parity of these pair amplitudes. The even-parity (odd-parity) pair amplitude should satisfy the relation $f_{i\pm}(\omega_n, \theta) = f_{i\mp}(\omega_n, -\theta)$ ($f_{i\pm}(\omega_n, \theta) = -f_{i\mp}(\omega_n, -\theta)$), with $i = 1, 2$. For an even(odd)-parity superconductor $\Phi_\pm(-\theta) = \Phi_\mp(\theta)$ ($\Phi_\pm(-\theta) = -\Phi_\mp(\theta)$). Then, we can show that $D_\pm(-\theta) = D_\mp(\theta)$ and $F_\pm(-\theta) = F_\mp(\theta)$ for even-parity case and $D_\pm(-\theta) = -D_\mp(\theta)$ and $F_\pm(-\theta) = -F_\mp(\theta)$ for odd-parity case, respectively.