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The size-dependent vibration analysis of micro-plates based on a modified couple stress theory

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ABSTRACT

A microscale vibration analysis of micro-plates is developed based on a modified couple stress theory. The presence of the length scale parameter in this theory enables us to describe the size effect in micro-structures. A variational approach based on Hamilton's principle is employed to obtain the governing equations of motion. To illustrate the new model, the free vibration analysis of a rectangular micro plate with two opposite edges simply supported and arbitrary boundary conditions along the other edges and a circular micro-plate are considered. The natural frequencies of micro-plates are presented for over a wide range of length scale parameters, different aspect ratios and various boundary conditions for both rectangular and circular micro-plates. The effect of length scale parameter on natural frequencies of micro-plates are discussed in details and the numerical results reveal that the intrinsic size dependence of material leads to increase the natural frequency.

1. Introduction

As the thickness of the general structures such as beams, plates and shells becomes on the order of microns, the size dependence effect of material make an important role in mechanical analyses of such structures. Due to the vast experimental expenses of micro-structures analyses, there is a great interest in applying continuum mechanics for analysis of micro-structures and explaining the size effects.

Using a higher order continuum theory, the classical couple stress elasticity was originated by Cosserat and Cosserat [1], Toupin [2], Mindlin and Tiersten [3]. They used two constants in order to account the size effects in isotropic materials. Yang et al. [4] developed an additional equilibrium relation to govern the behavior of couples. This useful work reduced the two independent higher order material length scale parameters to only one and formed a modified couple stress theory. Lim and He [5] studied the size-dependent geometrically nonlinear response of thin elastic films with nano-scale thickness based on a continuum approach. The film was assumed elastically isotropic, and Kirchhoff's hypothesis was adopted to approximate the deformation kinetics. Park and Gao [6] developed a model for bending of a Euler-Bernoulli beam by using a modified couple stress theory. A general thin plate theory including surface effects, which can be used for static and dynamic analysis of plate-like thin film structures, was proposed by Lu et al. [7]. The dynamic problems of simply supported Euler–Bernoulli beams were solved analytically by Kong et al. [8] on the basis of the modified couple stress theory. Ma et al. [9] considered a microstructure-dependent Timoshenko beam model for bending and vibration analysis of the simply supported beams by using a variational formulation. Tsiatas [10] studied static analysis of isotropic micro-plates using Kirchhoff plate model. Adopting Kirchhoff's theory of plates, Lazopoulos [11] investigated the bending of strain gradient elastic thin plates in order to determine the size effect. Asghari et al. [12] studied the size-dependent static and vibration analyses of micro-beams made of functionally graded materials on the basis of the modified couple stress theory. Xia et al. [13] investigated the theoretical analysis of micro-beams and studied bending, postbuckling and free vibration analyses. Wang et al. [14] developed a microscale Timoshenko beam model based on the strain gradient elasticity theory to capture the size effect.

In this paper, the new model is presented for vibration analysis of rectangular and circular micro-plates using a modified couple stress theory. Unlike the classical plate theory, this model contains an internal material length scale parameter and can predict the size effect in micro-scale plates. A variational formulation based on Hamilton's principle is employed to obtain the governing equations of motion and corresponding boundary conditions. A levy-type solution is developed for an isotropic micro-rectangular plate with two opposite edges simply supported and arbitrary boundary conditions along the other edges. Also, a solution is obtained for a circular micro-plate with arbitrary boundary condition at circular edge. The natural frequencies are presented for both rectangular

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and circular micro-plates with different length scale parameters, various boundary conditions and some aspect ratios. The results reveal that the size length scale has a significant effect when the thickness of the micro-plate becomes small. These results for natural frequencies of micro-plates can serve as reference values for other numerical analysis.

2. Modified couple stress formulations

In classical elasticity theory, stress and strain energy depend on the strain tensor and should not depend on the rotation vector because of frame indifference. However, the gradient of the rotation vector may occur as an argument in the constitutive equations. Based on the modified couple stress theory, the strain energy density is a function of both strain and gradient of the rotation vector as [4]

$$U = 1/2\int_V (\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij})dV \quad (1)$$

where σ, ε, m and χ are, respectively, the Cauchy stress, green strain, deviatoric part of couple stress and symmetric curvature tensor which are defined as

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} \quad (2a)$$

$$\varepsilon_{ij} = 1/2(u_{i,j} + u_{j,i}) \quad (2b)$$

$$m_{ij} = 2\mu l^2\ \chi_{ij} \quad (2c)$$

$$\chi_{ij} = 1/2(\theta_{i,j} + \theta_{j,i}) \quad (2d)$$

where λ and μ are Lame's coefficients and l in Eq. (2c) is a length scale parameter and is mathematically the square root of the ratio of the modulus of curvature to the shear modulus. It can be seen that in this model only one length scale parameter is needed to recognize the behavior of the structures. This length scale parameter is a material property which carried with it all of the difference between classical and couple stress elasticity theories. This parameter is small in comparison with the body dimensions and therefore its influence might become important as dimensions of a body diminish to the order of the length scale parameter (l).

Also, the parameter θ in Eq. (2) is the rotation vector and can be expressed in term of displacement vector as

$$\theta_i = 1/2e_{ijk}u_{k,j} \quad (3)$$

where e_{ijk} is the permutation symbol.

3. Dynamic model of a micro-rectangular plates

Consider a rectangular plate with dimension $a\times b$ and uniform thickness h as shown in Fig. 1. The displacement field of this plate is assumed to be

$$u_1(x_1,x_2,x_3,t) = u_0(x_1,x_2,t) + x_3\psi_1(x_1,x_2,t)$$
$$u_2(x_1,x_2,x_3,t) = v_0(x_1,x_2,t) + x_3\ \psi_2(x_1,x_2,t)$$
$$u_3(x_1,x_2,x_3,t) = w(x_1,x_2,t) \quad (4)$$

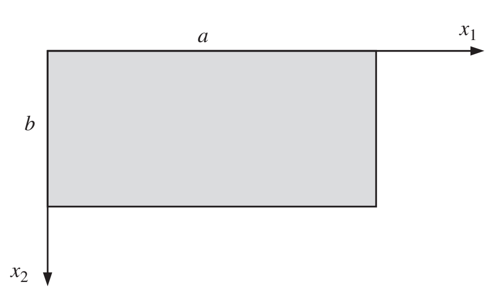

Fig. 1. Geometry of a rectangular micro-plate.

where u_0, v_0 and w are the displacement components of middle surface in x_1, x_2 and x_3 directions, respectively. According to the basic hypothesis of Kirchhoff plate theory, the rotation functions of the middle surface (ψ_1 and ψ_2) are considered as

$$\psi_1 = -w_{,1}$$
$$\psi_2 = -w_{,2} \quad (5)$$

where subscript comma denotes differentiate with respect to independent variables. Under the assumption of small deformation and linear strain–displacement relations, the strain components of a micro-plate can be expressed as

$$\varepsilon_{11} = u_{0,1} - x_3w_{,11} \quad \varepsilon_{12} = \varepsilon_{21} = 1/2(u_{0,2} + v_{0,1} - 2x_3w_{,12})$$
$$\varepsilon_{22} = v_{0,2} - x_3w_{,22} \quad \varepsilon_{13} = \varepsilon_{31} = \varepsilon_{23} = \varepsilon_{32} = \varepsilon_{33} = 0 \quad (6)$$

Also the rotation components and curvature components are obtained based on the Kirchhoff displacement components as

$$\theta_1 = w_{,2} \quad \theta_2 = -w_{,1} \quad \theta_3 = (v_{0,1} - u_{0,2})/2$$
$$\chi_{11} = w_{,12} \quad \chi_{12} = \chi_{21} = 1/2(w_{,22} - w_{,11}) \quad \chi_{13} = \chi_{31} = 1/4(v_{0,11} - u_{0,12})$$
$$\chi_{22} = -w_{,12} \quad \chi_{23} = \chi_{32} = 1/4(v_{0\ ,12} - u_{0,22}) \quad \chi_{33} = 0 \quad (7)$$

The governing equations of motion and corresponding boundary conditions can be obtained using Hamilton's principle, i.e.

$$H = \int_0^t [\delta T - (\delta U - \delta W)]\ dt \quad (8)$$

where U, W and T are strain energy, the work of external loads and kinetic energy of the micro plate, respectively. Considering the couple stress effect, the strain energy involves both classical and couple stresses terms as shown in Eq. (1). Expressing the strain and symmetric curvature tensors in terms of displacement components, the variation of the strain energy density of the micro-plate can be obtained as

$$\delta U = \int_V [\sigma_{11}(\delta u_{0,1} - x_3\delta w_{,11}) + \sigma_{22}(\delta v_{0,2} - x_3\delta w_{,22}) + \sigma_{12}(\delta u_{0,2} + \delta v_{0,1} - 2x_3\delta w_{,12})]dV + \int_V [m_{11}\delta w_{,12} - m_{22}\delta w_{,12} + m_{12}(\delta w_{,22} - \delta w_{,11}) + m_{13}(\delta v_{0,11} - \delta u_{0,12})/2 + m_{23}(\delta v_{0,12} - \delta u_{0,22})/2]dV \quad (9)$$

without the surface couple, the variation of work done by external forces in the form of transverse loading p and the variation of kinetic energy of the micro-plate may be expressed as

$$\delta W = \int_\Omega p\delta w d\Omega \quad (10a)$$

$$\delta T = \delta\int_V \frac{1}{2}\rho\left[(\ddot{u}_0 - x_3\ddot{w}_{,1})^2 + (\ddot{v}_0 - x_3\ddot{w}_{,2})^2 + \ddot{w}^2\right]dV \quad (10b)$$

where dot above each parameter denotes derivative with respect to time. Introducing Eqs. (9) and (10) into Hamilton's principle, the equations of motion of a rectangular micro-plate can be obtained as follows:

$$N_{11,1} + N_{12,2} + 1/2Y_{13,12} + 1/2Y_{23,22} = I_0\ddot{u}_0 - I_1\ddot{w}_{,1} \quad (11a)$$

$$N_{12,1} + N_{22,2} - 1/2Y_{13,11} - 1/2Y_{23,12} = I_0\ddot{v}_0 - I_1\ddot{w}_{,2} \quad (11b)$$

$$M_{11,11} + 2M_{12,12} + M_{22,22} - Y_{11,12} - Y_{12,22} + Y_{12,11} + Y_{22,12} - p = I_0\ddot{w} + I_1(\ddot{u}_{0,1} + \ddot{v}_{0,2}) - I_2(\ddot{w}_{,11} + \ddot{w}_{,22}) \quad (11c)$$