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The cyclotron resonance of impurity magnetopolarons in two-dimensional quantum dots for all coupling strengths

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ABSTRACT

Cyclotron resonance of magnetopolarons bound to a Coulomb impurity in a two-dimensional (2D) parabolic quantum dot (QD) is studied within a variational calculation for all coupling strengths. The Lee–Low–Pines–Huybrecht variational technique that was developed previously for all coupling strengths has been extended for polarons in a magnetic field. The dependence of the cyclotron resonance masses on the magnetic field, the confinement length, the electron–phonon coupling strength and the Coulomb binding parameter is investigated.

1. Introduction

With the advancement of nanofabrication technology, it is now possible to manufacture small zero-dimensional quantum dots (QDs) in which a few electrons can be confined in all the three dimensions spatially [1–3]. In such nanometer QDs, some novel physical phenomena and potential electronic device applications have generated a great deal of interest. The study of the impurity states in these low dimensional structures is an important aspect on which many theoretical and experimental works are based [4–6]. Recently, considerable effort [7–9] has been focused on exploring the polaron effect of QDs. The presence of a magnetic field makes the polaron problem in QDs more interesting since its existence means an additional confinement. Recently, Chen et al. [10] have studied the thickness dependence of the binding energy of an impurity bound polaron in a parabolic QD in magnetic fields using the second-order perturbation theory. Au-Yeung et al. [11] have investigated the combined effects of a parabolic potential and a Coulomb impurity on the cyclotron resonance of a three-dimensional (3D) bound magnetopolaron using Larsen's perturbation method. Zhu and Gu [12] have studied the cyclotron resonance of magnetopolarons in a parabolic QD with a strong magnetic field normal to the plane of the QD using the second-order Rayleigh–Schrodinger perturbation theory.

In the weak coupling case the states are extended and perturbation theory can be used, while in the other strong coupling limit, which corresponds to localized states, adiabatic methods are employed. Between them, variational techniques are adopted, where certain canonical transformations called Lee–Low–Pines (LLP) are performed and each of which has its own variational parameters. One of these transformations was modified by Huybrechts [13] in order to extend this approach to all coupling strengths. The LLPH method has been successfully carried out in calculation of the polaronic corrections to the ground- and first-excited states in QDs by Mukhopadhyay and Chatterjee [14].

The polaronic effects in QDs have also been examined by the Feynman–Haken path integral method [15] in the absence of a magnetic field. Recently, the related problem of an optical polaron bound to a Coulomb impurity in a QD has also been considered in the presence of a magnetic field [16–19]. By introducing a trial wave function constructed as a direct product form of an electronic part and a part of coherent phonons, Kandemir and Cetin [20] have investigated the polaronic effect on the low-lying energy levels of an electron bound to a hydrogenic impurity in a 3D anisotropic harmonic potential subjected to a uniform magnetic field. In the present paper our purpose is to study the cyclotron resonance of magnetopolarons bound to a Coulomb impurity in a two-dimensional (2D) parabolic QD. We shall employ the LLPH method to obtain the ground-state (GS) and excited-state (ES) energies of magnetopolarons embedded in 2D QD with parabolic confinement.

2. Theoretical model

We consider an electron, which is interacting with LO phonons in an isotropic harmonic potential and a magnetic field along the z direction. Suppose that we have a Coulomb impurity located at the origin of the QD. In the effective mass approximation the Hamiltonian of the electron–phonon system is given by

$$H=\frac{(\boldsymbol{p}+e\boldsymbol{A}/c)^2}{2m^*}+\frac{1}{2}m^*\omega_0^2\boldsymbol{\rho}^2+\sum_{\boldsymbol{q}}\hbar\omega_{LO}b_{\boldsymbol{q}}^+b_{\boldsymbol{q}}+\sum_{\boldsymbol{q}}(V_{\boldsymbol{q}}e^{i\boldsymbol{q}\cdot\boldsymbol{r}}b_{\boldsymbol{q}}+\mathbf{H.c.})-\frac{e^2}{\varepsilon_\infty\boldsymbol{r}} \quad (1)$$

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where $b_{\boldsymbol{q}}^{+}(b_{\boldsymbol{q}})$ is the creation (annihilation) operator of an optical phonon with a wave vector $\boldsymbol{q}$. ω_{LO} is the LO-phonon frequency, which is assumed to be dispersionless and ω_0 is the confinement frequency of the isotropic harmonic potential. V_q is defined as follows [21]:

$$V_{\boldsymbol{q}} = i(\hbar\omega_{LO}/q)(\hbar/2m^*\omega_{LO})^{1/4}(4\pi\,\alpha/V)^{1/2}) \tag{2}$$

where α is the electron–phonon coupling constant.

By using the symmetrical Coulomb gauge $\boldsymbol{A}=(-By/2,\ Bx/2,0)$ for the vector potential, Eq. (1) can be written as follows:

$$H = -\frac{\hbar^2\nabla_\rho^2}{2m^*} + \frac{1}{2}\omega_c L_z + \frac{1}{2}m^*\omega^2\boldsymbol{\rho}^2 + \sum_{\boldsymbol{q}}\hbar\omega_{LO}b_{\boldsymbol{q}}^{+}b_{\boldsymbol{q}} + \sum_{\boldsymbol{q}}(V_{\boldsymbol{q}}e^{i\boldsymbol{q}\cdot\boldsymbol{r}}b_{\boldsymbol{q}} + \mathbf{H}.\mathbf{c}.) - \frac{e^2}{\varepsilon_\infty \boldsymbol{r}} \tag{3}$$

where $\omega^2=\omega_1^2+(\omega_c/2)^2$ with the cyclotron frequency $\omega_c=eB/m^*c$.

In the LLPH method [13] the first LLP transformation is modified as

$$U_1 = \exp\left[-ia\sum_{\boldsymbol{q}}(q\cdot r b_{\boldsymbol{q}}^{+}b_{\boldsymbol{q}})\right] \tag{4}$$

where a is a variational parameter. Then after the second LLP transformation [22]

$$U_2 = exp\left[\sum_{\boldsymbol{q}}(f_q b_{\boldsymbol{q}}^{+} - f_q^* b_{\boldsymbol{q}})\right] \tag{5}$$

when $f_q(f_q^*)$ is the variational function, we obtain

$$H' = U_2^{-1}U_1^{-1}HU_1U_2 \tag{6}$$

When $a=1$ this modified procedure reduces to the LLP method, which should provide a description in the extended state limit, while for $a=0$ this approach is equivalent to the Landau–Pakar method [23], which is valid in the adiabatic limit and will be a useful approach in the localized state limit. Thus treating a as a variational parameter $(0<a<1)$ one can have a consistent theory encompassing the entire parameter space. The variational energy is now written as

$$E = \langle\phi|\langle 0|H'|0\rangle|\phi\rangle \tag{7}$$

where ϕ is the electronic function to be chosen variationally and $|0\rangle$ is the unperturbed zero phonon state. The variational energy then simplifies to

$$E = -\frac{\hbar^2}{2m^*}\langle\phi|\nabla_{\boldsymbol{\rho}}^2|\phi\rangle + \frac{1}{2}\omega_c\langle\phi|L_z|\phi\rangle + \frac{1}{2}m^*\omega^2\langle\phi|\boldsymbol{\rho}^2|\phi\rangle + \left[\sum_{\boldsymbol{q}}\left(\hbar\omega_{LO} + \frac{a^2\hbar^2q^2}{2m^*}\right)|f_q|^2\right] + \sum_{\boldsymbol{q}}(\xi_{\boldsymbol{q}}f_q^*\rho_q^* + \mathbf{H}.\mathbf{c}.) - \frac{e^2}{\varepsilon_\infty}\left\langle\phi\left|\frac{1}{r}\right|\phi\right\rangle \tag{8}$$

where

$$\rho_q = \langle\phi|e^{i(1-a)\boldsymbol{q}\cdot\boldsymbol{r}}|\phi\rangle \tag{9}$$

Minimizing E with respect to f_q^* now yields

$$f_q = -\frac{V_{\boldsymbol{q}}\rho_q^*}{\left(\hbar\omega_{LO} + \left(a^2\hbar^2q^2/2m^*\right)\right)} \tag{10}$$

and thus Eq. (8) reduces to

$$E = -\frac{\hbar^2}{2m^*}\langle\phi|\nabla_{\boldsymbol{\rho}}^2|\phi\rangle + \frac{1}{2}\omega_c\langle\phi|L_z|\phi\rangle + \frac{1}{2}m^*\omega^2\langle\phi|\boldsymbol{\rho}^2|\phi\rangle - \sum_{\boldsymbol{q}}\frac{|V_q|^2|\rho_q|^2}{\left(\hbar\omega_{LO} + \left(a^2\hbar^2q^2/2m^*\right)\right)} - \frac{e^2}{\varepsilon_\infty}\left\langle\phi\left|\frac{1}{r}\right|\phi\right\rangle \tag{11}$$

2.1. The ground state

We assume that the Gaussian function approximation is still valid in magnetic fields, where the GS electronic wave function is chosen as

$$\phi = \frac{\lambda}{\pi^{1/2}}e^{-\lambda^2\rho^2/2}|\xi(z)\rangle \tag{12}$$

with λ being a variational parameter to be determined.

Hence the GS energy expectation becomes

$$E_0 = \frac{\hbar^2}{2m^*}\lambda^2 + \frac{\hbar^2}{2\lambda^2m^*l_0^4} + \frac{m^*\omega_c^2}{8\lambda^2} - \sqrt{\frac{\pi}{2}}\hbar\omega_{LO}r_0\alpha\left(\frac{1+t\lambda}{t}\right)(1+\hbar e^{t^2}erfc(t)) - \beta\sqrt{\pi}\lambda \tag{13}$$

where $r_0=[\hbar/2m^*\omega_{LO}]^{1/2}$ is the polaron radius, $l_0=[\hbar/m^*\omega_0]^{1/2}$ is the effective confinement length of the QD and $t=(m^*\omega_{LO}/\hbar)^{1/2}(1-a)/(a\lambda)$ is to be treated as a new variational parameter instead of a. $\beta=e^2/\varepsilon_\infty$ being the Coulomb binding parameter.

Thus, the GS energy of this system can be obtained by minimizing Eq. (13) with respect to λ and t.

2.2. The first excited state

For the first ES we may choose the electronic wave function as

$$|\phi_{1\pm}\rangle = \frac{\lambda^2}{\pi^{1/2}}\rho\exp(-\lambda^2\rho^2/2)\exp(\pm i\phi)|\xi(z)\rangle|0_{ph}\rangle \tag{14}$$

which satisfies the following orthonormal relations:

$$\langle\phi_0|\phi_1\rangle = 0 \tag{15}$$

$$\langle\phi_1/\phi_1\rangle = 1 \tag{16}$$

Hence the first ES energy becomes

$$E_{1\pm} = \frac{\hbar^2}{m^*}\lambda^2 + \frac{\hbar^2}{\lambda^2m^*l_0^4} + \frac{m^*\omega_c^2}{4\lambda^2} \pm \frac{1}{2}\hbar\omega_c - \hbar\omega_{LO}r_0\alpha(1+t\lambda)\left(\frac{5}{16} + \frac{t^2}{8} - \frac{\sqrt{\pi}}{32t}(e^{t^2}erfc(t)(4t^4+12t^2+11) - 11)\right) - \frac{1}{2}\beta\sqrt{\pi}\lambda \tag{17}$$

Thus, the ES energy of this system can be obtained by minimizing Eq. (17) with respect to λ and t.

The renormalized cyclotron mass is defined to be

$$m_{\pm}^* = \frac{\omega_c m^*}{\omega^*} \tag{18}$$

which depends on the relevant cyclotron resonance frequency

$$\omega^* = \frac{E_{1\pm} - E_0}{\hbar} \tag{19}$$

3. Numerical results and discussion

The numerical results of the renormalized cyclotron mass for an impurity atom with the electron–phonon interaction in a parabolic QD versus the electron–phonon coupling constant, the applied