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Frequency-dependent Hall effect in spintronic systems under zero magnetic field ☆

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Abstract

It is shown that in an electronic system with finite Rashba coupling and in the absence of external magnetic field, the Hall resistivity (ρ_{xy}) is finite at both zero and finite frequencies. This Hall resistivity is determined by the reactive part (real part) of the inverse dielectric functions. This allows us to probe the real part of the dielectric function in a spintronic system by using a transport measurement.

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1. Introduction

The study of spin-electronics (or spintronics) has attracted tremendous attention in recent years, both in theoretical and experimental circles [1], thanks to the discovery of the long-lived (~100 ns) coherent electron spin states in n-typed semiconductors [2–7]. Spintronic systems and devices can be realized on the basis of diluted magnetic semiconductors and narrow-gap semiconductor nanostructures. In typical spintronic systems, electron states with opposite spin acquire different phase factors during their propagation in the presence of Rashba spin–orbit coupling [8]. New spin-related transport phenomena have been investigated, e.g. the spin and charge Hall effect [9,10].

So far all works on electron transport in spintronic systems are limited to cases where the applied field is static. For systems with finite spin splitting, transitions between different spin states due to photon scattering can play a significant role in the electrical transport. In this work, we report a theoretical investigation of frequency-dependent electrical transport in a system with finite Rashba coupling and under zero magnetic field. We are able to show, from the first principle, that spin-splitting due to the Rashba coupling can qualitatively change the electrical transport. For an isotropic system in the absence of Rashba coupling and external magnetic field, the current flows only along the direction of the applied electric field and the off-diagonal conductivity (or Hall conductivity) σ_{xy} equals zero. For a spintronic system with finite Rashba coupling, a finite electric current can flow in the direction perpendicular to the applied electric field. Furthermore, the real part of the Hall conductivity which is related to the Hall resistivity, ρ_{xy}, is determined by the reactive (real) part of the inverse dielectric function. All other resistivity components under zero or finite magnetic field in any electronic system are directly determined by the absorptive (imaginary) part of inverse dielectric function [11]. Therefore, an electronic system with finite Rashba coupling represents the first physical system we knew whose resistivity component is determined by $\Re e[1/\varepsilon(q,\omega)]$. This would allow us to use transport measurement to probe the reactive part of the dynamical dielectric function.

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2. Equation of motion for the density matrix

We consider a two-dimensional electronic system in the x–y plane in narrow-gap semiconductor nanostructures (e.g. InGaAs/InAlAs quantum wells). The Hamiltonian of a free electron is given as

$$H_0 = \frac{1}{2m^*}(p_x^2 + p_y^2) + \frac{\lambda}{\hbar}(\sigma_y p_x - \sigma_x p_y), \tag{1}$$

where m^* is the electron effective mass. The wave function can be written in the form of $\psi(x,y) = u_{\mathbf{k}}(x,y)\xi$ where $u_{\mathbf{k}}(x,y) = \exp(\mathrm{i}k_x x + \mathrm{i}k_y y)$. The eigenvalue is

$$E_\alpha^{(0)} = \frac{\hbar^2 k^2}{2m^*} + \alpha\lambda k, \tag{2}$$

where $k = \sqrt{k_x^2 + k_y^2}$ and $\alpha = \pm 1$, and eigenfunctions ξ_α are

$$\xi_{k,\alpha} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -\alpha(k_y - \mathrm{i}k_x)/k \end{pmatrix}. \tag{3}$$

We now construct electron quantum field operator, $\widehat{\Psi}(x,y) = \sum_{k,\alpha}\widehat{a}_{\mathbf{k}\alpha}u_{\mathbf{k}}(x,y)\xi_{\mathbf{k}\alpha}$, and $\widehat{\Psi}^\dagger(x,y) = \sum_{k,\alpha}\widehat{a}^\dagger_{\mathbf{k}\alpha}u^*_{\mathbf{k}}(x,y)\xi^*_{\mathbf{k}\alpha}$, where $\widehat{a}_{\mathbf{k}\alpha}(\widehat{a}^\dagger_{\mathbf{k}\alpha})$ is the creation (annihilation) operator for an electron with wave number $\mathbf{k}$ and Rashba state α.

For a many-electron system in the presence of electron–impurity scattering, the total Hamiltonian can be written in the form of $H = H_0 + H_{e-e} + H_{\mathrm{i}m}$, where H_0 is the Hamiltonian for a free electron with the spin–orbit interaction, H_{e-e} is the electron–electron interaction, and $H_{\mathrm{i}m}$ is the interaction between the electron and singly charged random impurity.

The electron motion can be described in two equivalent notations, the α- and the σ-notations. In the σ-notation, the single-electron density matrix element between states $\langle\sigma, k, t|$ and $|\sigma', k+p, t\rangle$ is defined as $F_{\sigma\sigma'}(\mathbf{k}+\mathbf{p}, \mathbf{k}, t) = \langle\widehat{a}^\dagger_{\mathbf{k},\sigma'}(t)\widehat{a}_{\mathbf{k}+\mathbf{p},\sigma}(t)\rangle$. Here σ and σ' can be up(u) or down(d) and the matrix element has four components. Under an external electrical field $\mathbf{E}(t) = \mathbf{E}\mathrm{e}^{-\mathrm{i}\omega t}$, the equation of the motion for the single-electron density matrix is found in a matrix form as

$$\begin{aligned}\left(\widehat{K}(\mathbf{k},\mathbf{p},\partial/\partial t) + \frac{e\hbar^2}{m^*c}\mathbf{p}\cdot\mathbf{A}(t)\right)F(\mathbf{k}+\mathbf{p},\mathbf{k},t) \\ = \sum_{\sigma,\mathbf{q},\mathbf{k}'} 2V(q)\left[F_{\sigma\sigma}(\mathbf{k}'+\mathbf{q},\mathbf{k}',t) + Ze^2\sum_i \mathrm{e}^{-\mathrm{i}\mathbf{q}\cdot\mathbf{R}_i}\right] \\ \times [F(\mathbf{k}+\mathbf{p},\mathbf{k}+\mathbf{q},t) - F(\mathbf{k}+\mathbf{p}-\mathbf{q},\mathbf{k},t)],\end{aligned} \tag{4}$$

where $\mathbf{A} = \mathbf{E}\mathrm{e}^{-\mathrm{i}\omega t}/(\mathrm{i}\omega)$, $\mathbf{p}, \mathbf{q}$ are electron momenta, $V(q) = 2\pi e^2/q$ is the Coulomb potential in two dimensions, $\mathbf{R}_i$ is the position of ith impurity, and $ZV(q)$ is the strength of electron impurity interaction. The density matrix is a four component vector given as,

$$F(\mathbf{k}+\mathbf{p},\mathbf{k},t) = \begin{pmatrix} F_{uu}(\mathbf{k}+\mathbf{p},\mathbf{k},t) \\ F_{ud}(\mathbf{k}+\mathbf{p},\mathbf{k},t) \\ F_{du}(\mathbf{k}+\mathbf{p},\mathbf{k},t) \\ F_{dd}(\mathbf{k}+\mathbf{p},\mathbf{k},t) \end{pmatrix}, \tag{5}$$

and $\widehat{K}(\mathbf{k},\mathbf{p},\partial/\partial t)$ is a 4×4 matrix whose matrix elements are

$$\widehat{K}\left(\mathbf{k},\mathbf{p},\frac{\partial}{\partial t}\right) = \begin{pmatrix} \eta & \lambda k_- & -\lambda k'_+ & 0 \\ \lambda k_+ & \eta & 0 & -\lambda k'_+ \\ -\lambda k'_- & 0 & \eta & \lambda k_- \\ 0 & -\lambda k'_- & \lambda k_+ & \eta \end{pmatrix},$$

where $k_\pm = k_y \pm \mathrm{i}k_x$, $\mathbf{k}' = \mathbf{k} + \mathbf{p}$ and $\eta = \hbar^2(p^2 + 2\mathbf{p}\cdot\mathbf{k})/2m^* - \mathrm{i}\hbar\partial/\partial t$. From $\widehat{K}(\mathbf{k},\mathbf{p},\partial/\partial t)$ we found $\Xi(\mathbf{k},\mathbf{p},\omega) = \widehat{K}^{-1}(\mathbf{k},\mathbf{p},\omega)$, $\widehat{K}(\mathbf{k},\mathbf{p},\omega)$ is the Fourier transformation of $\widehat{K}(\mathbf{k},\mathbf{p},\partial/\partial t)$.

3. The electrical current

From the definition of current operator, $\mathbf{J} = c\delta H/\delta\mathbf{A}$, we obtain the electrical current operators, $\mathbf{J}(\omega) = \mathbf{J}_1(\omega) + \mathbf{J}_0(\omega)$, where $\mathbf{J}_0(\omega) = (\mathrm{i}ne^2/m^*\omega)E_x = \sigma_0\mathbf{E}$. Here, n is the two-dimensional electron concentration. The current $\mathbf{J}_0$ is pure imaginary and thus does not contribute to the resistivity. The current $\mathbf{J}_1$ is complex and its average value is given in term of electron density matrices.

$$\begin{aligned}J_{1,x(y)}(\omega) = \frac{e\hbar}{m^*}\sum_{\mathbf{k}\sigma} k_{x(y)}F_{\sigma\sigma}(k,\mathbf{k},\omega) \\ \pm \mathrm{i}\frac{e\lambda}{\hbar}\sum_{\mathbf{k}}[F_{ud}(\mathbf{k},\mathbf{k},\omega) \mp F_{du(ud)}(\mathbf{k},\mathbf{k},\omega)].\end{aligned} \tag{6}$$

Following the method we developed previously [12], we have solved the Eq. (4) to the second order of electron–impurity interaction (up to Z^2) while the electron–electron interaction is treated in the mean-field approximation [13].

For narrow-gap semiconductor-based systems, the value of the dimensionless Rashba parameter $\Lambda = \lambda k_{\mathrm{F}}(0)/E_{\mathrm{F}}(0)$ is around $0.01-0.2$. Therefore, it is justified to neglect terms of order $0(\Lambda^4)$. In this case the conductivity tensor is given as

$$\begin{aligned}\hat{\sigma}(\omega) = \sigma_0 - \mathrm{i}\frac{Z^2ne^2}{m^{*2}\omega^3}\int\frac{\mathrm{d}\mathbf{q}}{2\pi}\mathbf{q}\mathbf{q}\left[\frac{V(q)}{\epsilon(q,0)} - \frac{V(q)}{\epsilon(q,\omega)}\right] \\ + \frac{Z^2ne^2\lambda^2}{\pi m^*\omega^4}\int\mathrm{d}\mathbf{q}\frac{\mathbf{q}(\hat{z}\times\mathbf{q})}{\varepsilon(q,0)}\left[\frac{V(q)}{\varepsilon(q,0)} - \frac{V(q)}{\varepsilon(q,\omega)}\right],\end{aligned} \tag{7}$$

where ϵ is the dielectric function with the Rashba coupling and ε is the dielectric function in the absence of the Rashba coupling. For the case where the electrical field is along the x-axis, the first term in Eq. (7) is σ_{xx} and second term is the Hall conductivity (σ_{yx}). σ_{xx} only depends