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Added mass of a spherical cap body

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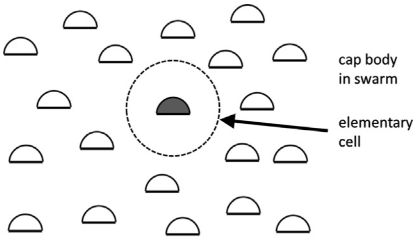
HIGHLIGHTS

- Dispersed flow modeling.
- Added mass coefficient for a cap-body.
- Collective added mass in a swarm of cap bodies.

GRAPHICAL ABSTRACT



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ABSTRACT

The added mass coefficient *C* was determined for a single spherical-cap body moving in a uniform unbounded fluid. An approximate simple physical model for *C* was suggested and was well compared with the analytical result of Kendoush, which likely is the only available theoretical result in the literature, up to date. The correct result for *C* was obtained via direct numerical flow simulation with CFD. Both the rigid and deformable (bubble, drop) cap body was considered. An approximate model was suggested for the collective added mass in a swarm of spherical cap bodies. A relation was found between the added mass of an unbounded cap body and a bounded spherical body. Practical explicit correlation formulas for *C* were obtained, suitable for engineering modelling of multiphase flow systems with bubbles, drops and solids. A relation between the added mass, Darwin drift, and fluid mixing was also noted.

1. Introduction

The goal of this contribution is to obtain information about the added mass (AM) coefficient *C* of a spherical-cap body moving in a uniform unbounded fluid. The reason is that this problem has not been paid sufficient attention, on the academic side. On the practical side, cap shaped bodies are common in many engineering applications dealing with dispersed multiphase systems (bubbles, drops, solids). Besides the article of Kendoush (2003, 2004), we could not find other contributions on the added mass of a cap body. The present study follows our two previous contributions on investigation into various area of the AM problem (Simcik et al., 2008; Simcik and Ruzicka, 2013).

It is known (e.g. Batchelor, 1967) that the inertia reaction of a fluid with respect to an accelerating submerged body can be expressed by the inertia mass tensor $\underline{\mathbf{M}}$. It relates the vector $\mathbf{a}$ of the body acceleration to the vector $\mathbf{F}$ of the inertia force exerted by the fluid on the body (inertial resistance or reaction):

$$\mathbf{F} = -\underline{\mathbf{M}} \times \mathbf{a}. \tag{1.1}$$

The mass tensor can be expressed as $\underline{\mathbf{M}} = \rho V \underline{\mathbf{C}}$, where ρ is the fluid density, V is the body volume, and $\underline{\mathbf{C}}$ is the inertia coefficient tensor. In the case with sufficient symmetry, the tensor $\underline{\mathbf{C}}$ becomes

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a mere scalar C. The added mass coefficient C says how much fluid would experience the same acceleration as the body, as if it would stick to it and move with it at the same speed. This virtual 'added volume of fluid' is expressed in units of the body volume V, see Fig. 1a. The total kinetic energy transmitted by the body into the whole fluid, which may be difficult to evaluate, equals the kinetic energy carried by this virtual added volume of fluid, which is easy to evaluate:

$$E=(1/2)\rho VCU^2 \tag{1.2}$$

The added mass effect of a single rigid spherical body in an unbounded fluid is $C_0=0.5$ (e.g. Lamb, 1932). This can be viewed as if the body moves in a vacuum but brings with it an extra carriage of the fluid amounting one-half on its volume V, traveling with the same speed U.

It is tempting to relate the concept of AM to the engineering problem of fluid mixing. One can argue that the added fluid volume given by C really moves with the body from one place to the other, which displacement causes the mixing. While this kind of reasoning is wrong, the result is correct (false implies anything).

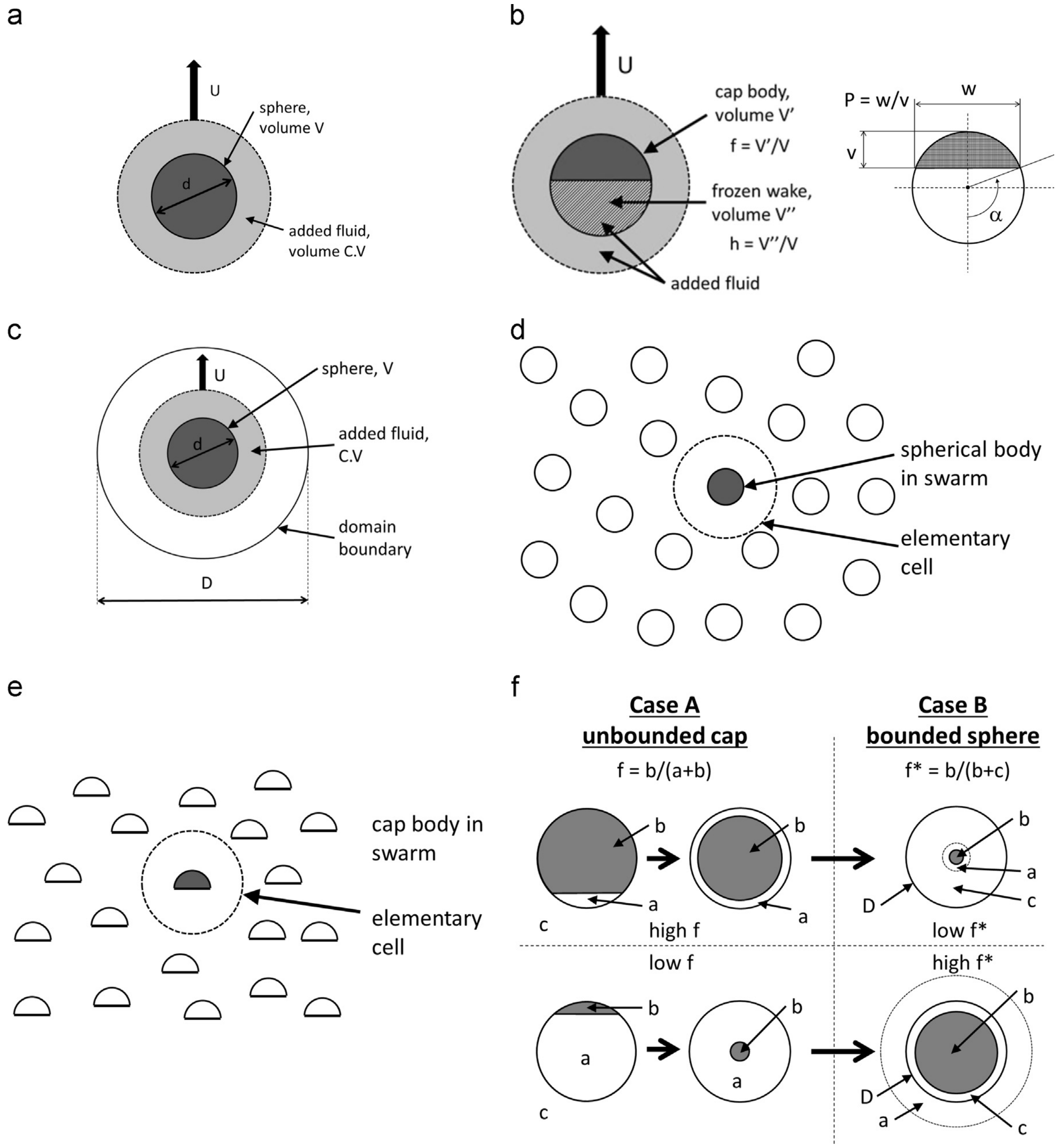

Fig. 1. (a) A spherical body (size d, volume V) moves at speed U in an unbounded flow domain. The added mass can symbolically be seen as an added fluid (volume $C\times V$) moving with the body at the same speed U. (b) Approximate model for the cap-shaped body: definition sketch. The sphere of the original body—spherical envelope (volume V) is divided into two parts: the cap body (volume V') and the added fluid seen as a 'frozen wake' (volume V''). The cap fraction is $f=V'/V$ and the liquid wake fraction is $h=V''/V$, with $f+h=1$. The cap-shape can be defined by the fraction f (or h), by the cap angle alpha, and also by the aspect ratio P=w/v (excentricity). (c) Bounded spherical body—definition sketch. A spherical body (size d) moves at speed U in a bounded spherical domain (size D). The added mass can symbolically be seen as an added fluid (volume $C\times V$) moving with the body at the same speed U. (d) A hypothetical 'elementary cell' (size D) around a test spherical particle (size d) in a dispersion of spherical particles with volumetric particle concentration $f^*=(d/D)^3$. (e) A hypothetical 'elementary cell' (size D) around a test cap-shaped particle in a dispersion of cap-shaped particles with volumetric particle concentration f^*. (f) Symmetry relation between Eq. (2.4) and Eq. (4.2): Case A—cap body in unbounded domain, Case B—spherical body in bounded domain. Schematic notation: a—added fluid, b—body, c—surrounding fluid. Case A has factor $f=b/(a+b)$ while Case B has factor $f^*=b/(b+c)$.