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Oscillation dynamics of sessile droplets subjected to substrate vibration

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HIGHLIGHTS

- Used high speed imaging to study sessile droplets excited by support vibrations.
- Studied the effect of driving amplitude and frequency on oscillation level.
- Modal analysis on the droplet shape through proper orthogonal decomposition.
- Duffing oscillator analogy established for a sessile droplet for the first time.

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ABSTRACT

Sessile droplets on a vibrating substrate are investigated focusing on axisymmetric oscillations with pinned contact line. Proper orthogonal decomposition is employed to identify the different modes of droplet shape oscillation and quantitatively assess the droplet oscillation and spectral response. We offer the first experimental evidence for the analogy of an oscillating sessile droplet with a non-linear spring mass damper system. The qualitative and quantitative agreement of amplitude-response and phase-response curves and limit cycles of the model dynamical system with that observed experimentally suggest that the bulk oscillations in the fundamental mode of a sessile droplet can be very well modeled by a Duffing oscillator with a hard spring, especially near the resonance. The red shift of the resonance peak with an increase in the glycerol concentration is clearly evidenced by both the experimental and predicted amplitude response curves. The influence of various operational parameters such as excitation frequency and amplitude and fluid properties on the droplet oscillation characteristics is adequately captured by the model.

1. Introduction

Droplets deforming on a solid surface are encountered in many industrial applications; lab-on-a-chip applications, ink-jet printing, pesticide coating, spray cooling, combustor walls, to name a few. Due to the vibratory environment existing in many of these applications, the efficiency of these processes, at the elementary level, depends crucially on the stability of the droplets adhered onto the substrate. The growth of droplet shape oscillations induced by the vibration of the substrate or during droplet impact can ultimately lead to the disintegration of a droplet into finer droplets. Droplet oscillations are also exploited to enhance internal mixing within the droplet in microfluidic applications (Mugele et al., 2006), to achieve high-speed focusing with liquid lenses (López and Hirsa, 2008) and also to measure fluid properties such as viscosity and surface tension (Egry et al., 2005).

Droplet oscillations have been the subject of extensive investigation in the past. In 1879, Lord Rayleigh (Rayleigh, 1879) formulated the expression for the frequency of natural oscillations of isolated inviscid droplet. Many studies have been carried out since to shed light on the various aspects of the oscillations exhibited by the free surface of droplets (see Ref. Milne et al. (2014) and references therein for a review). Here we discuss a few relevant studies on oscillations of sessile droplets. Theoretically, Strani and Sabetta (1984) discovered that for a droplet supported by a concave spherical bowl with pinned contact line, there exists a low-frequency mode that is not feasible for a free droplet. Lyubimov et al. (2006) presented a theoretical study on the axisymmetric and non-axisymmetric modes of oscillation of a hemispherical inviscid droplet on an oscillating surface with and without slip of the contact line. They found out that in the mobile contact line mode, the natural frequency of a sessile droplet coincides with the eigen frequency of a free spherical drop.

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On the experimental side, Noblin et al. (2004) observed that contact angle hysteresis leads to the oscillatory stick–slip motion of the triple line of large sessile drops at high amplitudes of oscillation. In addition to single-tone excitation, other techniques such as a white noise vibration or a momentary impulse have also been used to excite sessile droplets at multiple frequencies. By studying sessile droplets excited by white noise vibration, Mettu and Chaudhury (2012) reported that the contact line motion dampens the higher frequency modes more severely compared to the lower frequency ones. Sharp (2012) studied the effects of contact angle and viscosity on the resonance frequency by studying the standing waves on the free surface of sessile drops perturbed by a puff of air. Many of these studies concentrate on the linear oscillations of the droplet or on the contact line dynamics. However, as reported by Wilkes and Basaran (1997) pointed out, the finiteness of the forcing amplitude can lead to non-linearities in the oscillations of supported droplets.

The major aim of the present study is to offer experimental evidence for non-linear effects in the oscillation dynamics of sessile droplets. We also attempt to develop a simple model to simulate the response of a supported droplet, based on the analogy between the droplet and a driven spring–mass–damper system. While this kind of analogy has been applied in the case of isolated/constrained droplets subjected to aerodynamic forces (O'Rourke and Amsden, 1987; Esposito et al., 2010) or pendant droplets undergoing small-scale oscillations (Meier et al., 2000), it has never been attempted in the case of sessile droplets undergoing finite amplitude oscillations due to a vibrating support. Even though, based on the results from their numerical model (Wilkes and Basaran, 1997) and pendant droplet experiments (DePaoli et al., 1995), Basaran and co-workers indicated the analogy between a supported droplet and a Duffing oscillator, no attempt was made to develop such a one-dimensional non-linear model.

We adapt the well-known Duffing equation to a dynamical system that is excited by the support motion and thereby simulate the experimentally observed asymmetric nature of the droplet response curves, a characteristic of hard-spring nonlinearity, in the lowest mode of oscillation. The model accurately predicts the experimentally observed reduction in the height and increase in the width of the resonance peak, in addition to the red-shift of the resonance peak, with increasing fluid viscosity. Our way of presenting the droplet response through amplitude and phase response curves clearly demonstrates that the red shift is due to viscosity effects and not surface tension effects. proper orthogonal decomposition (POD) is employed to clearly elucidate the modal and spectral behavior of the interfacial waves on the droplet. Finally, in the interest of readers who seek a simple and rapid implementation of a one-dimensional model, a linear model is presented which simulates the observed asymmetric resonance peak of the droplet as accurately as the non-linear model, but by using a two-regime fit parameter in the inertia term of the resonator.

2. Experimental setup and methodology

The substrate was made of aluminum and was mounted on an electrodynamic shaker. High surface finish grinding was carried out to ensure that the mean roughness of the substrate is less than 0.5 μm. A droplet of volume 6 ± 1 μl was deposited using a micro-pipette at the center of the substrate so that the vertical axis of vibration of the shaker passes through the center of mass of the droplet. Between the trials the surface was rinsed with acetone, followed by water. Then the surface was blow-dried with dry air. Also it was ensured that the contact angles of the undisturbed droplets for a certain fluid did not vary substantially for the multiple runs. The amplitude and frequency of the substrate oscillation was fixed by controlling the magnitude and frequency of the voltage applied across the electromagnetic coil of the shaker. The droplet dynamics was captured at 2000 frames per second using a Photron Fastcam SA5 camera fitted with a microscopic zoom lens to achieve a spatial resolution of 5–9 μm per pixel. The spatial resolution was fixed for a given trial depending upon the extent of deformation exhibited by the droplet. The droplet was back illuminated using a white light source coupled to an optical diffuser. The video acquisition was initiated after the decay of the initial transient oscillations. The experiments were conducted at a controlled room temperature of 25 °C. To study the effect of fluid properties on the droplet oscillation characteristics, test fluids used were water and glycerol–water mixtures at glycerol volume fractions of 40%, 60% and 90% (see Table 1 for their room temperature properties (Cheng, 2008; Khossravi and Connors, 1993)).

For measuring the different characteristics of droplet oscillation, an in-house MATLAB code was developed to convert the grayscale images of the droplet to binary images by setting a threshold value on the pixel intensity. Using edge detection techniques, the droplet contact points were identified and eventually the 2-D projection of the spherical cap shape of the droplet was extracted from each image. The contact angle (defined as the macroscopic angle between the tangent to the droplet surface at the contact point and the substrate surface) in the unperturbed state for all the droplets studied, as measured through the image analysis, is observed to fall in a range of $72 \pm 7^\circ$; the variation being due to contact angle hysteresis (CAH) and the slight variation in surface tension among the fluids. Though theoretically the contact angle of a drop on a dry surface is single-valued, the droplet in practical situations exhibit a range of contact angle, called CAH, due to the existence of many metastable states because of the spatial heterogeneity in the chemical and topological features of the substrate (Neumann and Good, 1972). Note that the contact angle reported here is measured after the droplet reaches a steady shape subsequent to its deposition on the surface and hence will fall within the range of advancing or receding angles.

3. Results and discussion

Image analysis shows that the temporal profile of the displacement of the vibrating platform, y_s is purely sinusoidal (see Fig. 1 for a typical case at $f = 100$ Hz), i.e.,

$$y_s = Y_s \cos \omega t, \quad (1)$$

Table 1
Thermophysical properties of test fluids at 25 °C. The overhat denotes the quantity relative to that of water.

Fluid	Density, ρ (kg m^{-3})	Dynamic viscosity, μ (mPa s)	Surface tension, σ (N m^{-1})	$\hat{\rho}$	$\hat{\mu}$	$\hat{\sigma}$	Driving frequency range, f (Hz)	Substrate amplitude range, Y_s (mm)
Water	996.85	0.89	0.072	1	1	1	10–300	0.01–1.2
40% Glycerol	1117.5	4.06	0.068	1.12	4	0.94		
60% Glycerol	1169.6	12.81	0.0667	1.17	14	0.93		
90% Glycerol	1239.3	208.71	0.064	1.24	234	0.89		