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Multicritical points of the Potts spin glasses on the triangular lattice

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Abstract

Conjecture on the exact multicritical points for several types of the Potts spin glass model on the triangular lattice is given, using a technique based on the duality in conjunction with the replica method. The considered Potts spin glass models are of two types; only with three-body interactions and with two- and three-body interactions on the triangular lattice. In the former case, the multicritical points are predicted for the general q-state Potts spin glass model, while in the latter case the results are presented for two types of the 2-state Potts spin glass.
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1. Introduction

New information processing using the quantum states, namely the quantum information, is rapidly developing now. One of the issues is to develop strategies to eliminate the defects of instable quantum states by the surrounding environment. The quantum error-correcting code is one of the strategies. Its concept is to remove errors in the qubit systems with the aid of redundancy. It was pointed out that the toric code, which is one of the quantum error-correcting codes, has a relationship with the phase transition in spin glass systems [1]. More precisely, the accuracy threshold of the toric code can be estimated from the location of the multicritical point, which lies on the special subspace, the Nishimori line [2,3], in the phase diagram of some spin glass models. Thus it is important to determine the location of the multicritical point for not only an intrinsic interest in itself but also the practical purpose of evaluating the efficiency of the toric code. Though less exact methods for the spin glass models are known, an approach to determine the exact location of the multicritical points is recently developing now; conjecture on the exact location of the multicritical point [4,5]. The conjecture has given many results, which are in positive agreement with the results by other approaches; the $\pm J$ Ising model on the square, triangular and hexagonal lattices and so on [6–8]. However, its validity is still not proved. The result of the prediction of the multicritical point will be desired for its reliability in estimating the accuracy threshold of the toric code. We therefore need to prove the validity of the conjecture, and clarify the bound applicable to use the conjecture.

We propose, in this paper, two types of the Potts spin glass models with two- and three-body interactions on the triangular lattice. In this case, two parameters of their interactions exist. Thus it is possible to predict many multicritical points with various combinations of the parameters of their interactions. Such multicritical points provide us with useful and helpful examples, which enable us to extensively verify the validity of the conjecture by other approaches.

2. Pure model and its duality

It will be helpful to first review the q-state Potts model with two- and three-body interactions on the triangular lattice with no randomness and its duality [9]. The duality is one of the useful techniques to determine the transition point. We will use the duality to predict the location of the multicritical point.

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We write spin variables as ϕ_i and differences as $\phi_{ij} \equiv \phi_i - \phi_j$, which take integer values from 0 to $q-1 \pmod q$. Then the partition function of the Potts model with two- and three-body interactions is defined through the following face Boltzmann factor on elementary up-pointing triangles,

$$A_{\triangle}^{K_2,K_3}[\phi_{12},\phi_{23},\phi_{31}] \equiv \exp\left\{K_2\sum_{i\neq j}\delta(\phi_{ij}) + K_3\prod_{i\neq j}\delta(\phi_{ij})\right\}, \quad (1)$$

where $i\neq j$ runs over the pairs on the bonds (12), (23) and (31) surrounding each elementary up-pointing triangle. K_2 and K_3 are coupling constants of the two- and three-body interactions.

Though the usual duality changes the triangular lattice into the hexagonal lattice, a direct duality transformation for the triangular lattice without recourse to the hexagonal lattice was established by simultaneous use with star–-triangle transformation [9]. It is shown, by this direct triangular duality, that the Potts model with two- and three-body interactions is the self-dual model satisfying with the following duality relations:

$$v_2^* = \frac{qv_2}{y}, \quad y^* = \frac{q^2}{y}, \quad (2)$$

where $v_2 = \exp(K_2) - 1$ and $y = \exp(K_3 + 3K_2) - 3\exp(K_2) + 2$. The fixed condition for the duality is given by $y = q$. This condition determine the phase boundary in the range of Wu–Zia criteria $J_3 + 3J_2 > 0$, and $J_3 + 2J_2 > 0$ [10].

3. Potts spin glass

Next we consider the Potts spin glass, adding to randomness the previously introduced Potts model. It is defined through the following face Boltzmann factor:

$$A_{\triangle}^{K_2,K_3}[\phi_{12},\phi_{23},\phi_{31}] = \exp\left\{K_2\sum_{i\neq j}\delta(\phi_{ij}+l_{ij}) + K_3\prod_{i\neq j}\delta(\phi_{ij}+m_{ij})\right\}, \quad (3)$$

where ϕ_{ij} are the same symbols as in the previous section. l_{ij} are the random variables for the two-body interactions, and m_{ij} are for the three-body interactions with the following distribution functions:

$$P(l_{ij}) = \begin{cases} 1-(q-1)p_2 & l_{ij}=0 \\ p_2 & l_{ij} = \text{the other} \end{cases} = p_2\exp\{K_{p2}\delta(l_{ij})\}, \quad (4)$$

$$P(\{m_{ij}\}) = \begin{cases} 1-(q^3-1)p_3 & \{m_{ij}\} = \{0,0,0\} \\ p_3 & \{m_{ij}\} = \text{the other} \end{cases} = p_3\exp\left\{K_{p3}\prod_{i\neq j}\delta(m_{ij})\right\}, \quad (5)$$

where $\exp(K_{p2}) = \{1-(q-1)p_2\}/p_2$ and $\exp(K_{p3}) = \{1-(q^3-1)\,p_3\}/p_3$. It is possible to calculate the exact internal energy in a special subspace, defined by $K_2 = K_{p2}, K_3 = K_{p3}$, as shown in Refs. [2,3]. The subspace is called the Nishimori line. However, in our model, two independent variables exist. It is therefore called here the Nishimori "surface". It is expected that there are intersections between the Nishimori surface and a phase boundary, namely a multicritical "line".

Another type of the Potts spin glass is considered by slightly modified random variables $\{m_{ij}\} \to \{l_{ij}\}$ in Eq. (3). It is also possible to calculate the exact internal energy on the Nishimori surface $K_2 = K_{p2}, K_3 = K_{p3}$. We call the former Potts spin glass model type-I, and the latter one type-II. Two slightly different spin glass models are equal to each other for the case only with three-body interactions and only with two-body interactions, and the way how we calculate and analyze is equivalent between types I and II.

4. Conjecture on multicritical points

Ensuring the existing successful cases, we need the following conditions to derive the multicritical points:

(i) On the Nishimori line or surface, the self duality should be satisfied at least for the $n=1$ and 2 replicated systems.

We concentrate, on the system, on the Nishimori line, where it is believed that the multicritical point lies [2,3].

(ii) Transition points for their replicated systems are determined only by a single equation:

$$\tilde{A}_n^*[\{0\},\{0\},\{0\}] = \tilde{A}_n[\{0\},\{0\},\{0\}], \quad (6)$$

where $\tilde{A}_n$ is the replicated and configurational-averaged face Boltzmann factor and $\tilde{A}_n^*$ is its dual.

In the case of the pure systems, it is satisfied with the equation similar to Eq. (6), which expresses when neighboring spin variables take equal values between an original and its dual Boltzmann factors. It is also expected that Eq. (6) represents the location of the singularity of the replicated system. The validity for this hypothesis is still not proved, so the hypothesis is a conjecture.

We examined the self-duality for $n=1$ and 2 replicated models of the type-I and type-II Potts spin glasses. It is confirmed that the q-state Potts spin glasses only with three-body interactions satisfy self-duality when replica number n is 1 and 2. It is also found that the 2-state Potts spin glass with two- and three-body interactions is self-dual for the replica number 1 and 2; both of type-I and type-II. However, the general q-state Potts spin glasses with two- and three-body interactions ($q \geqslant 3$, integer) are not. Therefore, it would be possible to obtain the multicritical points of the q-state Potts spin glasses only with three-body

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