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Letters

Surface effects on static bending of nanowires based on non-local elasticity theory

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Abstract

The surface elasticity and non-local elasticity effects on the elastic behavior of statically bent nanowires are investigated in the present investigation. Explicit solutions are presented to evaluate the surface stress and non-local elasticity effects with various boundary conditions. Compared with the classical Euler beam, a nanowire with surface stress and/or non-local elasticity can be either stiffer or less stiff, depending on the boundary conditions. The concept of surface non-local elasticity was proposed and its physical interpretation discussed to explain the combined effect of surface elasticity and non-local elasticity. The effect of the nanowire size on its elastic bending behavior was investigated. The results obtained herein are helpful to characterize mechanical properties of nanowires and aid nanowire-based devices design.

Keywords: Surface effect; Non-local elasticity; Nanowires; Size effect; Euler–Bernoulli beam

1. Introduction

Outstanding mechanical properties of nanowires have been of considerable interest to researchers. For example, Wu et al. [1] measured the yield strength of Au nanowires by three-point bending using atomic force microscopy (AFM), and it's average values are 5.6 ± 1.4 GPa, which is more than 25 times higher than the bulk Au values. Treacy et al. [2] found that the Young's modulus of carbon nanotubes was in the Tera-Pascal (TPa) range. Cuenot et al. [3] reported the diameter-dependent elastic modulus effects in Ag and Pb nanowires. Meanwhile, the classical beam theory has been unsuccessful to theoretically analyze the mechanical properties of one-dimensional nano-materials. Hence, accurate description of nanowires' mechanical behavior is essential.

Surface effects have been recognized as significant factors during the deformation process of nanobeams. Chen et al. [4] proposed a core–shell composite nanowires model to explain the surface effects on the mechanical behavior of nanowires. He et al. [5] investigated surface stress and surface elasticity effects on the elastic behavior of statically bent nanowires. Jiang et al. [6] addressed combined surface and shear deformation effects based on the Timoshenko beam theory and the Young–Laplace equation. Wang and Feng [7] studied surface effects on buckling and vibration behavior of nanowires. All these research reports show that the surface effects play a significant role in the deformation behavior of one-dimensional nano-materials.

Based on the lattice dynamics theory and experimental observations on phonon dispersion, Eringen [8,9] proposed the non-local elasticity theory in 1972. According to this theory, it is assumed that the stress at a given reference point depends not only on the strain at this point, but also on the strain at other points in the body. This way, the influence of the long range forces between the atoms is taken into consideration, and thus the internal size scale can be introduced in the constitutive equations. In recent years, many researchers have successfully applied the non-local elasticity theory for explaining the deformation behavior of micro- and nanobeams [10–13].

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In this letter, the non-local elasticity theory is implemented to analyze the bending behavior of centrally loaded nanowires with consideration of surface elasticity and surface stress.

2. Non-local elasticity and surface effects

Under certain conditions, based on the non-local elasticity theory, the non-local stress tensor, σ_{xy}, within a two-dimensional region, using the Green's functions, is expressed as [8]:

$$\sigma'_{xy} = \left[1-(e_0 l)^2\nabla^2\right]\sigma_{xy}, \tag{1}$$

where σ'_{xy} is the (classical) stress tensor, l represents internal characteristic length (e.g., the lattice parameter, grain size, C–C bond length, etc.). The Laplace operator ∇^2 equals $\partial^2/\partial x^2+\partial^2/\partial y^2$ in Cartesian coordinates, and e_0 is a constant appropriate to each material. Eringen [8] obtained the magnitude of e_0=0.39 by matching the dispersion curves of the plane waves with those of atomic lattice. Hence, the Hooke's law for uniaxial stress state can be expressed as:

$$\sigma(x)-(e_0 l)^2\frac{\partial^2\sigma(x)}{\partial x^2} = E\epsilon(x) \tag{2}$$

Since the surface-to-volume ratio is large in nano-materials, nanowires were treated as a superposition of the surface layers and the bulk volume. The thickness of the beam is much larger than the thickness of the surface layer t_0. This way, the traditional flexural rigidity D for the bulk material is replaced by the effective flexural rigidity $D*$ for the composite beam. The effective flexural rigidity $D*$ for either rectangular or circular cross-section is:

$$D* = \begin{cases} \frac{Eab^3}{12}+\frac{E_s ab^2}{2}+\frac{E_s b^3}{6} & \text{(rectangle)} \\ \frac{\pi E d^4}{64}+\frac{\pi E_s d^3}{8} & \text{(circular)} \end{cases}, \tag{3}$$

where a is the length of rectangle, b represents the width of rectangle, d is the diameter of circular, E and E_s represent the Young's modulus of the bulk and the surface, respectively.

The existing constant residual surface tension on the surfaces above and below the bulk material causes a nanobeam to curve. The mathematic relation between the curvature tensor κ and the stress jump $\left\langle \tau_{ij}^{+}-\tau_{ij}^{-}\right\rangle$ across a surface is based on the generalized Laplace–Young equation [5,6,14]:

$$\left\langle \tau_{ij}^{+}-\tau_{ij}^{-}\right\rangle n_i n_j = \tau_s\kappa, \tag{4}$$

where τ_{ij}^{+} and τ_{ij}^{-} denote the upper and the lower surface stresses, respectively, n_i is the unit normal vector to the surface, κ is the curvature tensor of the nanowire and τ_s is the surface stress tensor given by [5,6,15]:

$$\tau_s = \tau_0 + E_s\varepsilon_x, \tag{5}$$

where τ_0 is the residual surface stress along the longitudinal direction of the nanobeam and ε_x is the strain along the nanowire longitudinal direction.

According to Eq. (4), the stress jump leads to a distributed transverse force $q(x)$ along the nanowire longitudinal direction [14]. For a deformed nanowire, the distributed force is given by [5,6]:

$$q(x) = Hw''(x), \tag{6}$$

where $w(x)$ denotes the nanobeam transverse displacement, and H is a constant parameter given by [5,6]:

$$H = \begin{cases} 2\tau_s a & \text{(rectangle)} \\ 2\tau_s d & \text{(circular)} \end{cases} \tag{7}$$

3. Non-local elasticity and surface stress coupling effects on the Euler–Bernoulli beam

Considering the Euler–Bernoulli beam model, the equilibrium equations for the shear force, T, the bending moment, M, and the transverse distributed load, $q(x)$, are:

$$\frac{\partial T}{\partial x}+q(x)=0 \tag{8}$$

$$T-\frac{\partial M}{\partial x}=0 \tag{9}$$

The bending moment constitutive relation accounting for the non-local elasticity and surface stress effects is written as:

$$M-(e_0 l)^2\frac{\partial^2 M}{\partial x^2} = -D^*\frac{\partial^2 w}{\partial x^2} \tag{10}$$

In view of Eqs. (8)–(10), the governing equation for the bending of non-local Euler–Bernoulli beam with the surface effects is given by

$$D^*\frac{\partial^4 w}{\partial x^4}+(e_0 l)^2\frac{\partial^2 q(x)}{\partial x^2}-q(x)=0 \tag{11}$$

By substituting Eq. (6) into Eq. (11), one obtains

$$\left[D^*+H(e_0 l)^2\right]\frac{\partial^4 w}{\partial x_4} = H\frac{\partial^2 w}{\partial x^2} \tag{12}$$

Letting

$$\eta_{ns}^{E} = \frac{HL^2}{D^*+H(e_0 l)^2} \tag{13}$$

The boundary conditions for the two kinds of the end are:

$$\text{Clamped end}: w(0)=0, w'(0)=0 \tag{14}$$

$$\text{Simply supported end}: w(0)=0, w''(0)=0 \tag{15}$$

Fig. 1 shows the deformation of a nanobeam with surface stress in different boundary conditions. As a constant concentrated force P is loading the free end at $x=L$, the moment and the force equilibrium conditions of the clamped-free beam (C-F) are:

$$-M(0) = PL+\int_0^L Hw''(x)x dx = PL+HLw'(L)-Hw(L) \tag{16}$$

and

$$T(0) = P+\int_0^L Hw''(x)dx = P+Hw'(L)-Hw'(0), \tag{17}$$

respectively. When the simply supported beam (S-S) subjected to a concentrated force P at the midpoint $x=L/2$, the slope at $x=L/2$