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# Taylor dispersion in oscillatory flow in rectangular channels

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## HIGHLIGHTS

- The effect of side walls on dispersion in oscillatory flows was explored.
- For small frequency, the dispersion coefficient approaches that of Poiseuille flow.
- For large frequency, the dispersion coefficient scales as $Pe^2/\Omega^2$.
- At small frequencies, the 2D model can significantly underestimate the dispersion.

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## ABSTRACT

This paper focuses on exploring the effect of the side walls on dispersion in oscillatory Poiseuille flows in rectangular channels. The method of multiple time scales with regular expansions is utilized to obtain analytical expressions for the effective dispersivity $D^*_{3D}$. The dispersion coefficient is of the form $D^*_{3D}/Pe^2 = f(\Omega \equiv \omega h^2/D, Sc \equiv D/\nu, \chi \equiv w/h)$ where $Pe \equiv <u> h/D$, $<u>$ is the root mean square of the cross-section averaged velocity, $\omega$ is the angular velocity, $2w$ and $2h$ are, respectively, the width and the height of the cross-section, $D$ is the solute diffusivity, and $\nu$ is the fluid kinematic viscosity. The analytical results are compared with full numerical simulations and asymptotic expressions. Also effect of various parameters on dispersion coefficient is explored. For small oscillation frequency $\Omega$ the dispersion coefficient approaches the time averaged dispersion of the Poiseuille flow and for large $\Omega$, $D^*_{3D}$ scales as $Pe^2/\Omega^2$ where $Pe = <u> h/D$. Due to its relative simplicity, the 2D model is frequently utilized for calculating dispersion in channels. However at small dimensionless frequencies the 2D model can significantly underestimate the dispersion, particularly for channels with large $\chi$. At large $\Omega$ the dispersion coefficient predicted from the 2D model becomes reasonably accurate, particularly for channels with large $\chi$. For a square channel, the 2D prediction is reasonably accurate for all frequencies. The results of this study will enhance our understanding of transport in microscale systems that are subjected to oscillating flows, and potentially aid technological advances in diverse areas relevant to microfluidic devices.

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## 1. Introduction

Geometries in which the axial length scale is much longer than the transverse length scales are common both in biological, such as blood flow, and industrial, such as microfluidics, applications (Ghosh et al., 2009; Gregory et al., 2007; Lee et al., 2008; Thomas and Narayanan, 2001). On time scales larger than those for transverse diffusion, the concentration in large aspect ratio flows depends only on time and the axial direction. However the transverse concentration gradients on the short time scale combined with the axial stretching lead to a much larger spread of an injected pulse compared to just diffusion. The spreading of the pulse as it convects with the mean velocity is characterized by the dispersion coefficient (Brenner and Edwards, 1993). Taylor (Taylor, 1953) first obtained the dispersion coefficient for Poiseuille flow in a tube and showed that the dimensionless dispersion coefficient is $Pe^2/48$. Here Péclet number is defined as $Pe = UR/D$ where $U$ is average velocity, $R$ is radius of channel and $D$ is the molecular diffusivity. Aris (Aris, 1960) extended Taylor's

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results by using the method of moments to show that the dispersion coefficient for flow in a two-dimensional channel is $Pe^2/210$. Here Péclet number is defined as $Pe = Uh/D$ when $h$ is the height of channel. Doshi et al. (1978) extended Aris's results to rectangular channels and showed that that the dispersion coefficient for a rectangular channel reaches a limit as the cross-section aspect ratio approaches infinity, but this limiting value is about eight times that for the dispersion coefficient for the 2D channel. Purtell (Purtell, 1981) explored oscillatory flow in a tube and Watson (Watson, 1983) obtained a general expression for dispersion in oscillatory flows for any arbitrary cross-section. Watson's results can be used to obtain explicit expressions for a circular pipe and a 2D channel. Explicit analytical expressions for dispersion in a rectangular channel have not been reported for oscillatory flows but such flows are becoming increasingly relevant in microfluidics applications such as DNA amplification through temporal temperature gradients, microfluidic oscillators and magnetic field actuated microfluidic valves. In such applications it is critical to accurately predict the spread of an injected pulse to avoid contamination due to mixing between two successive samples, and so here we focus on developing analytical expressions for dispersion in oscillatory flows in a channel. The microfluidic flows typically contain periodic oscillation superimposed on the unidirectional flow, but Tripathi et al. Tripathi et al. (2005) showed that dispersion coefficient in such flows is simply additive and thus here we consider flow driven by a purely sinusoidal pressure with zero mean velocity.

In this paper, we have developed a 3D analytical solution for dispersion of oscillating flow by doing a multiple time scale analysis. There are several important time scales in this problem, including those of the axial convection as well as transverse and axial mass diffusion and period of oscillation. The main quantity of interest is however the dependence of the cross-section average of the concentration on the longest time scale, which corresponds to the axial convection. The multiple time scale analysis is thus suitable for analyzing this problem. The framework developed here could be adapted to other problems, including problems with transverse flow or fields such as the field–flow fractionation. The results presented here, including the analytical expressions and also the numerical results on the effect of various parameters on dispersion such as frequency, Womersley number and aspect ratio, could also be very useful to chip designers for multiple purposes such as oscillating microreactor or bio-assays such as DNA amplification.

## 2. Theoretical approach

### 2.1. Velocity field of oscillatory flow

The oscillatory flow in large aspect ratio channels has an extra time scale of oscillation in addition to the other time scales corresponding to axial convection and transverse and axial diffusion. So the method of multiple time scales is a natural choice for exploring this problem. Bender and Orszag (1978) To take advantage of the disparities in various length and time scales, we solve the problem by assuming regular perturbation expansions in $\varepsilon$, which is defined as the ratio of the total height ($2h$) to the axial length ($2L$), i.e., $\varepsilon = h/L$. The height is by definition the smaller of the two sides of the cross-section so total width ($2w$) is larger than $2h$. The geometry is illustrated in Fig. 1. The origin of the coordinate system is chosen to be the center of the channel with $x$-axis in the direction of the flow, $y$-axis perpendicular to the flow along the height direction and $z$-axis perpendicular to the flow along the width direction. The end effects and effects of initial conditions on flow are neglected and only fully developed oscillating flow of a viscous incompressible fluid is considered. The flow is driven by a position-independent time periodic axial pressure gradient, i.e.

$$\frac{\partial p}{\partial x} = \beta \cos(\omega t) = \mathrm{Re}(\beta e^{i\omega t}) \tag{1}$$

where $\beta$ and $\omega$ are the amplitude and the frequency of the oscillating pressure gradient, respectively. The velocity is considered to be uniaxial, i.e., the $y$ and $z$ components of the velocity are zero. Under these conditions, the $x$-component of the dimensionless Navier–Stokes equation becomes

$$\frac{\partial u^*}{\partial \tau} = -\frac{1}{\Omega/Sc}\mathrm{Re}(e^{i\tau}) + \frac{1}{\Omega/Sc}\left(\frac{\partial^2 u^*}{\partial \xi_1^2} + \chi^2 \frac{\partial^2 u^*}{\partial \xi_2^2}\right) \tag{2}$$

where $u^* = u/(\beta h^2/\mu)$, $\tau = \omega t$, $\xi_1 = y/h$, $\xi_2 = z/w$, $Sc = \nu/D$, $\Omega = \omega h^2/D$ and $\chi = h/w$. In the above expressions, $D$ is the solute diffusivity; $\mu$ and $\nu$ are fluid's dynamic and kinematic viscosities, respectively.

The above differential equation is solved by expressing velocity in terms of Fourier series, i.e.

$$u^* = \mathrm{Re}\left(\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} u_{mn} \cos\left(\left(\frac{2n-1}{2}\right)\pi\xi_1\right)\cos\left(\left(\frac{2m-1}{2}\right)\pi\xi_2\right)e^{i\tau}\right) \tag{3}$$

Note that the above expansion automatically satisfies the no-slip boundary conditions at the channel walls. Substituting the Fourier expansion of the velocity in the dimensionless Navier–Stokes equation (2) yields

$$\begin{aligned} & i\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} u_{mn} \cos\left(\left(\frac{2n-1}{2}\right)\pi\xi_1\right)\cos\left(\left(\frac{2m-1}{2}\right)\pi\xi_2\right) = \\ & -\tfrac{1}{\Omega/Sc} - \tfrac{1}{\Omega/Sc}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} u_{mn}\left[\left(\frac{2n-1}{2}\pi\right)^2 + \chi^2\left(\frac{2m-1}{2}\pi\right)^2\right]\cos\left(\left(\frac{2n-1}{2}\right)\pi\xi_1\right)\cos\left(\left(\frac{2m-1}{2}\right)\pi\xi_2\right) \end{aligned} \tag{4}$$

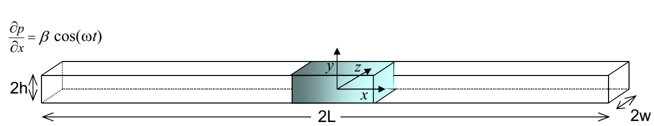


**Fig. 1.** Schematic diagram of the 3D microchannel with oscillating pressure.

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