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Numeration and type synthesis of 3-DOF orthogonal translational parallel manipulators

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Abstract

Low-degree-of-freedom (Low-DOF) parallel manipulators (PMs) have drawn extensive interests, particularly in type synthesis in which two main formal approaches were established by using the reciprocal screw system theory and Lie group theory. This paper aims at numeration and type synthesis of orthogonal translational parallel manipulators (OTPMs) by resorting to an integration of the group-based method and graphical representation of the topology. For this purpose, the concept of Cartesian DOF-characteristic matrix, originated from displacement subgroup and displacement submanifold, is proposed. A new approach based on the combination of the atlas of Cartesian DOF-characteristic matrix and the displacement group-theoretic method is addressed for both exhaustive classification and type synthesis of OTPMs. The proposed approach is prone to construct an orthogonal structure and is easy to realize the complete classification and exhaustive enumeration of this class of low-DOF PMs.

Keywords: Type synthesis; Displacement group; Orthogonal translational parallel manipulator

1. Introduction

Compared with conventional serial devices, parallel manipulators (PMs) have the merits of potential higher stiffness, lower inertia, larger payload/weight ratio, and better dynamic characteristics. But some disadvantages occur, especially in 6-DOF fully parallel manipulators. This is reflected in the complexity of the direct kinematics due to highly coupled position and orientation motions of the platform. In addition to this, manufacturing at a low cost and high accuracy is a challenge. However, fewer than 6 DOFs, or called low-DOF parallel manipulators [1], if properly designed, may alleviate or even overcome the above shortcomings. In particular, much effort has been dedicated to the design of 3-DOF translational PMs (TPMs) [2–9]. They received particular attention due to their vast industry applications as alternatives to traditional serial positioning systems, like as assembly manipulators, parallel kinematic machines (PKMs), and micromanipulators.

Although there are increasing designs for such a class of manipulators in the literatures, the mechanism types are still much fewer than those required. In this regards, a rigorous synthesis approach by the aid of mathematical tools appears appealing since it does address the difficulties encountered in the conceptual design of low-DOF PMs.

In the recent five years, there are increasing studies on the structural synthesis of low-DOF PMs in the literatures, also including TPMs. In particular, several systematic approaches have been proposed. They are the enumeration approach based on the general Chebyshev–Grübler–Kutzbach (CGK) mobility formula [10], the motion synthesis

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approach based on the displacement subgroup theory [11–17] or single-open-chain theory [18,19], the constraint synthesis approach based on the reciprocal screw system theory [20–30], and the inference or conversion approach [31,32] based on the classical mechanism makeup.

In a strict sense, the enumeration method belongs to number synthesis of mechanism and has its limitation [33]. Other than the motion synthesis method based on the displacement subgroups, which can describe the finite motion of kinematic pairs or kinematic chains, the constraint synthesis method based on the reciprocal screws generally falls into the category of instantaneous motion because both twists and wrenches are the elements of Lie algebra instead of smooth manifold. Whereas the method of inference originated from a known elemental or simple structure also proves effective in the construction of a new mechanism.

Although the above four approaches provide a systematic framework for the structural design of low-DOF PMs, the type synthesis of PMs with particular geometry which is required to fulfill some specified tasks, such as PMs with decouple motion [34,35] or the remote-center-of motion(RCM) PMs [36], or type synthesis of PMs with particular kinematic features including isotropy [37–40], weak motion sensitivity [41], and high rotational ability [42] is still a comparably difficult task.

This paper mainly aims at exploring a simple and effective synthesis procedure for a class special decouple-motion TPMs, i.e., the orthogonal TPMs as a complement to the above general methods. For this purpose, the concept of Cartesian DOF-characteristic matrix (CDM), originated from the displacement subgroup and displacement submanifold, is proposed. A new approach based on the combination of the atlas of Cartesian DOF-characteristic matrix (ACDM) and the displacement group theory is addressed for both exhaustive numerations and type synthesis of OTPMs. In order to verify the effectiveness of the proposed method, OTPMs with both symmetrical and asymmetrical architecture are synthesized accordingly.

2. Displacement subgroup and displacement submanifold

As found in Ref. [11], the set of 6-dimensional rigid motion can be endowed with the algebraic structure of a group, represented by D as Lie group. Any further motion of a rigid body can be described by a subset of D, which may be either a group, called a displacement subgroup (DSG) or a displacement submanifold (DSM). The set of allowed relative displacements between two rigid bodies in a given kinematic chain is called kinematic bond. A kinematic chain generating the bond is named mechanical generator of the bond. In addition, Hervé enumerated all 12 kinds of displacement subgroups of D. Note that these subgroups can be represented in two configurations: nominal and conjugated configurations. For example, both $T(z)$ and $T(\boldsymbol{w})$ are the same representations of one-dimensional translational subgroup, but the former represents translations along the direction z of Cartesian coordinate frame and the latter represents the movements in the direction of any axis $\boldsymbol{w}$. In this paper, the descriptions of these subgroups in their nominal configurations are commonly used due to their special representations and potential advantages.

Amongst these displacement subgroups, $R(N, z)$, $T(z)$, $H(N, z, p)$, $C(N, z)$, $G(z)$, and $S(N)$ are associated with six lower kinematic pairs: Revolute pair (R), Prismatic pair (P), Helical pair (H), Cylinder pair (C), Planar pair (E) and Spherical pair (S). All these six lower pairs, combined with some composite joints such as Universal pair (U), P_a and U^* [43], can be regarded as primitive generators of DSGs or DSMs. Additional six displacement subgroups addressed in reference [11] include identity subgroup ε, planar-translation subgroup $T_2(z)$, spatial translation subgroup T, translating-screw subgroup $Y(N, z, p)$, Schönflies subgroup $X(z)$, and Euclidean group D itself. Hervé also enumerated the composition and intersection of different displacement subgroups. The intersection of subgroups follows the rules of intersection of sets.

For a serial kinematic chain or a serial manipulator composed of rigid bodies 1, 2, …, $n-1$, n, the allowed displacements of body n relative to body 1(usually a fixed base) is a subset (DSG or DSM) of the group D, which is indeed a kinematic bond. This bond is generated by the composition by implementing the product of all displacement subgroups associated with the lower pairs in the kinematic chain. In a PM, however, the set of allowed rigid displacement of moving platform is obtained from the intersection of the kinematic bonds generated by all limb kinematic chains. Therefore, it is primarily necessary to recall some preliminaries of operations on DSGs/DSMs.

Lemma 1. *Assuming that both A and B are displacement subgroups of D, the product of these two subgroups* $A \cdot B$ *is generally not a DSG, but a DSM included in D. At the same time,* $A \cdot B$ *is not commutative in most cases, i.e.,* $A \cdot B \neq B \cdot A$. *On the contrary, if the product of these two DSGs can commute to each other, the resulting DSG reduces to an Abel group.*

Example 1. Both $R(N, z)$ and $T(z)$ are displacement subgroups of D, and their product $R(N, z) \cdot T(z)$ can commute to each other, i.e.

$$R(N, z) \cdot T(z) = T(z) \cdot R(N, z) \tag{1}$$

Therefore, $R(N, z) \cdot T(z)$ constitutes an Abel DSG and is indeed a cylindrical-motion displacement subgroup $C(N, z)$.

Lemma 2. *Assuming that both A and B are the displacement subgroups included in a common subgroup Q, i.e.,* $A \subseteq Q$, $B \subseteq Q$, *due to the product closure in a subgroup, the product of these subsets* $A \cdot B$ *must be included in the same subgroup, i.e.* $A \cdot B \subseteq Q$.