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Capillary torque on a rolling particle in the presence of a liquid film at small capillary numbers

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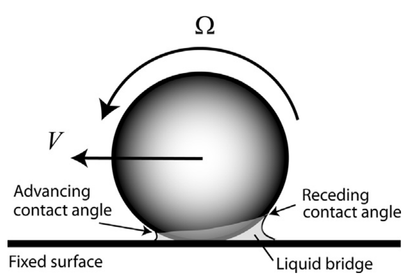
HIGHLIGHTS

- Two mechanisms analyzed for capillary torque on rolling particle with liquid film.
- One part of torque associated with rearward shift of liquid bridge.
- Second part associated with asymmetry of advancing and receding contact angles.
- Capillary torque is found to vary as a power-law with capillary number.
- Good agreement with experimental data for particle rolling down inclined slope.

GRAPHICAL ABSTRACT



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ABSTRACT

A theoretical analysis was developed for the capillary torque acting on a spherical particle rolling on a flat surface in the presence of a thin liquid film. The capillary number (the ratio of viscous force to surface tension force) is assumed to be sufficiently small that the liquid bridge has a circular cross-section. The theory identifies two mechanisms for capillary torque. The first mechanism results from the rearward shift of the liquid bridge in the presence of particle rolling, which causes the line of action of the pressure force within the liquid bridge to be located behind the particle centroid, inducing a torque that resists particle rolling. The second mechanism results from the contact angle asymmetry on the advancing and receding sides of the rolling particle, which leads to a net torque on the particle arising from the tangential component of the surface tension force. Estimates for these two types of capillary torque are obtained using experimental data, and correlations for both torques are obtained in the form of power-law fits as functions of the capillary number. When combined with a standard expression for viscous torque on a rolling particle, the capillary torque expressions are found to yield predictions for particle terminal velocity that are in good agreement with experimental data for a particle rolling down an inclined surface.

1. Introduction

When granular matter is exposed to moisture, even in very small amounts, such as through condensation from atmospheric humidity, the individual particles become surrounded by a thin liquid film. When these particles come into contact with each other or with a surrounding enclosure, the surrounding films of the contacting particles form a liquid bridge which gives rise to capillary forces that pull the particles together (often called the 'liquid bridge force'). Capillary forces often have a detrimental effect in granular flow applications since they act to promote caking between particles and to resist the free flow of particles under gravitational or other driving forces. Studies of the effect of moisture have been reported for granular flows of different types,

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including avalanche flows and granular piles (Bocquet et al., 2002; Halsey and Levine, 1998; Tegzes et al., 2002, 2003; Hornbaker et al., 1997; Nowak et al., 2005; Soria-Hoyo et al., 2009; Mason et al., 1999), motion of particles in a rotating drum (Liu et al., 2011, 2013; Brewster et al., 2009), segregation and mixing of particles of different sizes (Radl et al., 2010; Samadani and Kudrolli, 2000; Hsiau et al., 2008; Li and McCarthy, 2003), particles in a granular shear flow (Yang and Hsiau, 2006), and compaction of granular media (Fiscina et al., 2010).

The normal capillary force between particles, acting in a direction parallel to the line connecting the particle centroids, has been the subject of a large number of previous experimental and theoretical studies. The early literature on this topic primarily considered liquid bridge force and rupture criteria for static particles (Hotta et al., 1974; Mehrotra and Sastry, 1980; Lian et al., 1993; Willett et al., 2000). Models for normal force that account for the viscous force acting between particles with non-zero relative motion were developed by Ennis et al. (1990) and Matthewson (1988). A detailed experimental study by Pitois et al. (2000, 2001) compared several existing models for the liquid bridge force and rupture criteria to experimental data for both static and moving particles.

It was noted by Bico et al. (2009) that the thin liquid film surrounding a particle also has an effect on a particle rolling along a plane wall, giving rise to a capillary torque that resists the rolling motion. This capillary torque arises from the asymmetry of the liquid bridge that develops as a result of the rolling motion of the particle (Fig. 1). Bico et al. found that the shape of the liquid bridge varies as a function of capillary number $\mathrm{Ca}=\mu|V|/\sigma$, where μ is the liquid viscosity, σ is the liquid–gas surface tension, and V is the particle velocity along the surface. For $\mathrm{Ca}<1$, the liquid bridge has an approximately circular shape, whereas for $\mathrm{Ca}>1$ the liquid bridge forms a cusp with a trailing wake. An experimental study was reported by Schade and Marshall (2011) that used transparent lubricants with different viscosity values to examine a 13 mm diameter sphere rolling in a fixed position on a translated flat surface coated with a thin liquid film to obtain data for the change in contact angle and contact point location as functions of capillary number and Reynolds number.

A number of different effects act in concert to generate the torque on a rolling particle in the presence of a thin liquid film, including surface tension forces, pressure reduction within the liquid bridge, and viscous friction. This paper develops a theoretical expression for the capillary torque, which combines the first two effects listed above. After adding the viscous torque on a rolling particle to the capillary torque, an equation is obtained for the velocity of a particle rolling down an inclined planar surface at low values of capillary number. The results are compared to experimental data of Bico et al. (2009). The theoretical model for the normal force on a particle in the presence of a thin liquid film is reviewed in Section 2. Approximations made in the normal force theory form the foundation of the capillary torque theory discussed in Section 3. Section 4 gives predictions for velocity of a particle rolling down a slope in the presence of both viscous and capillary torque. Conclusions are given in Section 5.

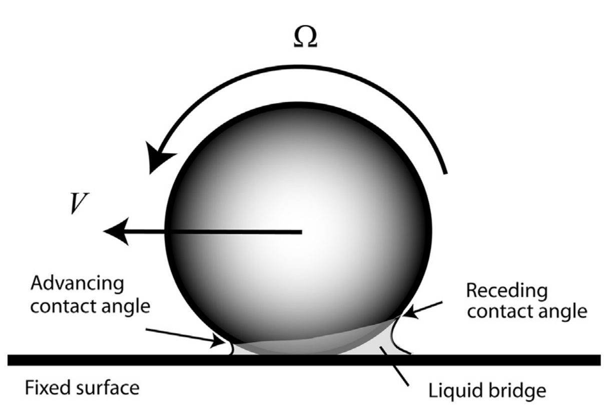

Fig. 1. Schematic diagram of a particle rolling on a flat surface.

2. Normal capillary force

We consider a spherical particle with radius r_p that is connected to a plane wall by an axisymmetric liquid bridge, with separation distance $h(t)$ between the particle and the wall, as shown in Fig. 2a. In terms of a cylindrical polar coordinate system (r,z), the radius of curvature of the liquid–gas interface in the r–z plane is denoted by ρ_1 and the radial position of the liquid bridge at the mid-plane between the sphere and the planar surface is ρ_2. The solid–liquid contact angle θ and the half-filling angle ϕ are defined as indicated in Fig. 2a.

In a static condition, an equal and opposite force acts on the particle and the wall which arises both from the component of the surface tension force oriented normal to the wall and from the pressure reduction within the liquid bridge that occurs as a result of the interface curvature. The sum of these two effects gives rise to an attractive force F_{cap} referred to as the capillary force, acting along the wall unit normal. The Young–Laplace formula gives the pressure reduction across the interface as

$$\Delta p_l=p_{liq}-p_{gas}=\sigma(\rho_1^{-1}-\rho_2^{-1}), \tag{1}$$

where σ is the liquid–gas surface tension and the negative sign in (1) results from the fact that the center of the tangent circles corresponding to ρ_1 and ρ_2 are on opposite sides of the interface. An approximate solution for the capillary force is obtained from the "gorge approximation" (Hotta et al., 1974; Lian et al., 1993), in which the force on the sphere is written in terms of the pressure force and surface tension force exerted on the midplane of the liquid bridge as

$$F_{cap}=2\pi\sigma\rho_2+\pi\rho_2^2\Delta p_l=\pi\sigma\rho_2\left[1+\frac{\rho_2}{\rho_1}\right], \tag{2}$$

where the last expression is obtained using (1).

In the case where the volume of the liquid bridge is small, the radial location $r=b$ of the liquid–solid–gas triple point satisfies $b\ll r_p$, and as a consequence we can conclude $\rho_2\ll r_p$ and $\phi\ll 1$. Since for small fill angles, elementary geometry can be used to show that the ratio of the radii of curvature satisfies $\rho_1/\rho_2=O(\phi)$, it follows that for small liquid volumes the force on the particle arising from the pressure reduction dominates the surface tension force, and the expression (2) for the capillary force reduces to leading order to

$$F_{cap}=\pi\sigma\rho_2^2/\rho_1 \tag{3}$$

The radius ρ_2 can be approximated for small liquid volumes as $\rho_2\cong b\cong(2r_ps)^{1/2}$, where b is the radial location of the liquid–gas–solid triple point on the sphere and s is the height of the triple point above the bottom of the sphere (Fig. 2b). A second geometrical consequence of the assumption of small fill angle is the approximation $2\rho_1\cos\theta\cong s+h$ for the radius ρ_1. Substituting these two approximations into (3) yields the capillary force as

$$F_{cap}=4\pi r_p\sigma\cos\theta\left(1+\frac{h}{s}\right)^{-1} \tag{4}$$

which is the same as the expression derived by Maugis (1987) using a thermodynamic approach related to crack propagation theory.