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On skirted drops in an immiscible liquid

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HIGHLIGHTS

- It is shown that the viscosity of the host fluid causes a tapering of the skirts sometimes seen trailing drops and bubbles.
- This tapering is an essential ingredient in explaining the origin of the finite length of skirts.
- Scalings of the skirt length and volume with the Reynolds, Eötvös and Morton numbers are suggested.

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ABSTRACT

Large drops rising or sinking in an immiscible liquid can develop thin trailing structures commonly referred to as "skirts". The paper describes a mathematical model for the thickness of these skirts accounting for the viscous boundary layer that develops along the surface of the parent drop and of the skirt itself. Unlike earlier theories, the skirt thickness is found to decrease with distance from the drop rim, which illuminates the mechanism which terminates the skirt at a finite length. A scaling of the skirt length is suggested by an analysis of published data, which also leads to a scaling for the volume of liquid in the skirt. The theoretical predictions are compared with the few experimental results for which sufficiently detailed information is available.

1. Introduction

A thin layer of fluid, commonly referred to as a "skirt", is sometimes observed issuing from the rim of large bubbles and drops translating in a sufficiently viscous liquid. An example of a numerical simulation illustrating these structures is given in Fig. 1a.

Skirts can grow to a stable length (see e.g. Guthrie and Bradshaw, 1969; Shoemaker and Marc de Chazal, 1969; Wairegi, 1974; Bhaga and Weber, 1981), become unstable and shed small fragments (see e.g. Wairegi, 1974; Bhaga, 1976), or undergo a stronger instability and shed larger volumes by the process shown in Fig. 1b which is referred to as exfoliation (Wairegi, 1974; Wairegi and Grace, 1976). The increased contact area between the two fluids that results from all these processes enhances the mass transfer coefficient, and this has been the primary reason for several past investigations of the phenomenon (Davenport et al., 1967; Calderbank et al., 1970; Guthrie and Bradshaw, 1973; Wairegi and Grace, 1976).

Another consequence of the phenomenon worth pointing out is that, when skirts become unstable and break up, drops (or bubbles) with a size much smaller than that of the parent drop (or bubble) can be generated. In the case of an underwater oil spill, for example, the reduced size of these fragments facilitates their transformation by the metabolic activity of small-scale marine life and even their dissolution (Hebert and Poulet, 1980; Almeda et al., 2013).

The first scientific report mentioning the skirt phenomenon appears to be a paper by Thomson and Newall (1885) who studied the impact and subsequent motion of drops falling in a variety of immiscible liquids. Most of the papers published in more recent times have been devoted to skirt formation by gas bubbles (Angelino, 1966; Davenport et al., 1967; Guthrie and Bradshaw, 1969; Calderbank et al., 1970; Hnat and Buckmaster, 1976; Bhaga and Weber, 1981). Comparatively fewer studies have been devoted to the case of drops. The results of Thomson and Newall (1885) were mostly qualitative. More quantitative information has been published by Shoemaker and Marc de Chazal (1969) and Wairegi and Grace (1976) and additional data can be found in the dissertations of Wairegi (1974) and Bhaga (1976).

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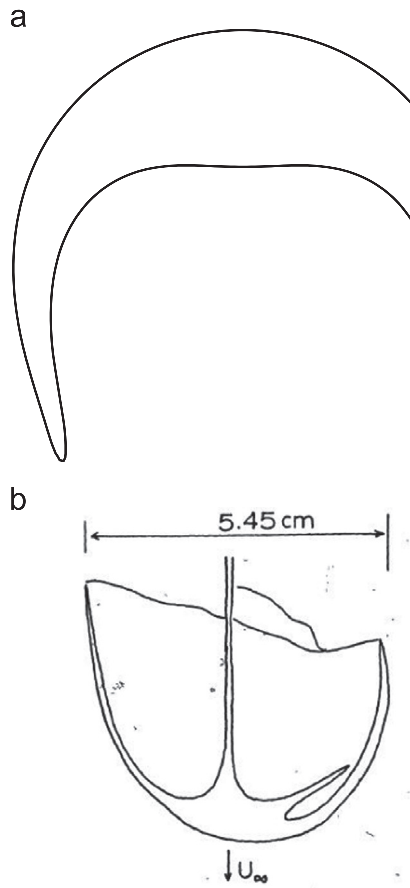

Fig. 1. (a) A frame from the numerical simulation of a two-dimensional drop rising in a viscous liquid; viscosity ratio $\mu_d/\mu_c=0.1$, density ratio $\rho_d/\rho_c=0.5$, Reynolds number Re=0.85, Eötvös number Eo=360, Morton number Mo=500. The total height of the drop in the image is 1.5 times the equivalent spherical diameter (courtesy of Y.H. Tseng). (b) Exfoliating process of the skirt formed by a silicon oil drop ($\mu_d=0.0465$ Pa s, $\rho_d=958$ kg/m^3) falling in a paraffin oil ($\mu_c=0.2$ Pa s, $\rho_c=883$ kg/m^3); the equivalent spherical diameter of the drop is $d_e=39.6$ mm, the fall velocity U=68.8 mm/s and the surface tension coefficient $\sigma=7$ mN/m; $\mu_d/\mu_c=0.233$, $\rho_d/\rho_c=1.085$, Re=12.03, Eo=165, Mo=4.4 (from Wairegi, 1974).

Skirt formation is one instance of phenomena in which small fluid masses break away from much larger ones by processes which do not depend in an obvious way upon the length scales characterizing the parent fluid. Phenomena of this type, such as hydrodynamic or electrohydrodynamic tip streaming, have been known for a very long time (see e.g. Zeleny, 1917; Taylor, 1934, 1964) but have recently acquired greater relevance due to the wide diffusion of electrospray mass spectrometry (see e.g. Fenn et al., 1989; Abonnenc et al., 2010; Wang et al., 2013) and the production of micro-drops in co-flowing fluid systems (Basaran, 2002; Suryo and Basaran, 2006; Barrero and Loscertales, 2007; Marin et al., 2009; Castro-Hernandez et al., 2012).

The primary purpose of the present paper is to develop a theory for the skirt thickness in liquid–liquid systems. Our theory is similar to earlier ones (Guthrie and Bradshaw, 1969; Wairegi, 1974), but differs from them in that we account for the viscous boundary layer that develops along the surface of the parent drop and of the skirt itself. A very significant consequence is that, unlike earlier theories, the skirt is found to thin in the downstream direction. As explained in Section 5, the length of the skirt is most likely due to this circumstance. The nature of the boundary layer is profoundly different for drops and bubbles, which makes our results only quantitatively applicable to the former. However, a qualitatively similar process would also occur in the case of bubbles.

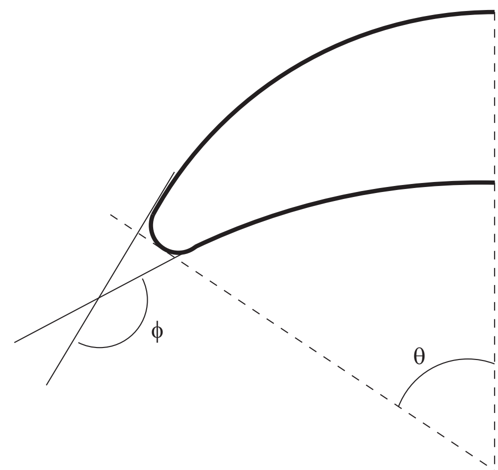

Fig. 2. Definition of the angles used in the conditions for skirt formation in Section 2.

Numerically, the prediction of skirted drops and bubbles represents a considerable challenge due to the very fine resolution made necessary by their small thickness. Several papers claim success in this endeavor by showing results in which the bubble develops a short downstream-directed rim or resembles an inverted cup with a gas layer of comparable thickness all around (Hua et al., 2008; Yu and Fan, 2008). These structures are really due to the pressure recovery on the downstream side of the drop or bubble and are therefore not true skirts, whose formation is dominated by viscosity and whose thickness is very much smaller than the thickness of the liquid or gas layer constituting the drop or bubble body. The recent work of Ohta and Sussman (2012) improves on these results, but it does not seem to yet reach the level of detail that is demonstrated by the best available work on tip streaming in low-Reynolds-number fluid dynamics (Suryo and Basaran, 2006; Zhou et al., 2006) or electro-hydrodynamics (Collins et al., 2008, 2013). Clearly, this is an area where further numerical work would be desirable. In particular, numerical simulations of the three-dimensional unsteady problem would help considerably to shed light on the fluid mechanics of these intriguing phenomena.

After an overview of the existing theories concerning the formation and thickness of skirts, we present our own version and compare its predictions with the available experimental data. We then turn to the problems of the skirt length and volume before summarizing our conclusions in the last section.

2. Mechanisms for skirt formation

The experimental observation that the appearance of skirts requires a sufficiently viscous continuous phase is at the basis of all existing theories of skirt formation. Bhaga (1976) used a local Stokes-flow approximation near the rim of the bubble (Fig. 2) to estimate the normal stress in the outer phase and balance it by the effect of surface tension. He argued that a skirt must form when such a balance becomes impossible and was led to the criterion

$$4K_B\frac{1+\cos\phi}{2\phi-\sin 2\phi}\frac{\mu_c U}{\sigma}\geq 1. \tag{1}$$

The angle ϕ is defined in Fig. 2, U is the rise velocity of the bubble, μ_c is the viscosity of the continuous phase (namely, the outer fluid) and σ is the coefficient of interfacial tension. The constant K_B accounts for the nature of the overall flow around the bubble and for the bubble shape. When the lower surface of the bubble is flat, $\phi=\pi-\theta$ with the angle θ defined in Fig. 2. Furthermore, if the