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An all-sky radiative transfer method to predict optimal tilt and azimuth angle of a solar collector

Christopher J. Smith [a,*], Piers M. Forster [b], Rolf Crook [a]

[a] *Energy Research Institute, School of Chemical and Process Engineering, University of Leeds, Leeds LS2 9JT, United Kingdom*
[b] *Institute for Climate and Atmospheric Science, School of Earth and Environment, University of Leeds, LS2 9JT, United Kingdom*

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Abstract

This paper describes a radiative transfer method for calculating radiances in all-sky conditions and performing an integration over the view hemisphere of an arbitrary plane to calculate tilted irradiance. The advantage of this method is the combination of cloud parameters inside the radiative transfer model with a tilt procedure. For selected locations this method is applied with cloud, ozone, water vapour and aerosol input data to determine tilted irradiance, horizontal irradiance and optimal tilt angle. A validation is performed for horizontal and tilted irradiance against high-quality pyranometer data. For 27 sites around the world, the annual horizontal irradiation predicted by our model had a mean bias difference of +0.56% and a root-mean-squared difference of 6.69% compared to ground measurements. The difference between the annual irradiation estimates from our model and the measurements from one site that provides tilted irradiance were within ±6% for all orientations except the north-facing vertical plane. For European and African sites included in the validation, the optimal tilt from our model is typically a few degrees steeper than predictions from the popular PVGIS online tool. Our model is generally applicable to any location on the earth's surface as the satellite cloud and atmosphere data and aerosol climatology data are available globally. Furthermore, all of the input data are standard variables in climate models and so this method can be used to predict tilted irradiance in future climate experiments.

Keywords: Radiative transfer; Tilt; Radiance; Cloud

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1. Introduction

The orientation of a plane solar collector such as a PV panel can be varied in the tilt and azimuth directions in order to maximise the incident irradiance. One way to accurately assess the solar resource available on a tilted plane and determine the optimum angle to orient a fixed angle PV panel in the real world, is to position pyranometers in several plane orientations and record the sum of irradiance over a sufficiently long period of time. In practice this is rarely completed, so models to predict the tilted irradiance are used.

There are two concepts fundamental to the method described. Firstly, cloud optical properties, from satellite retrievals, are integrated into the radiative transfer (RT) calculation. Secondly, tilted irradiance is derived from a surface radiance field. RT methods are frequently used to model clear-sky solar irradiance (Bird and Riordan, 1986; Gueymard, 1995; Mueller et al., 2004). Sometimes cloud effects are introduced as an adjustment to the clear-sky values depending on satellite-derived cloud albedo (Cano et al., 1986) or tuned based on observed historical ground-level irradiance (Nann and Emery, 1992). In other

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* Corresponding author. Tel.: +44 (0)113 343 2498.
E-mail address: pmcjs@leeds.ac.uk (C.J. Smith).

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studies cloud effects are included directly. Lohmann et al. (2006) used data from meteorological reanalyses and cloud parameters from the International Satellite Cloud Climatology Project (ISCCP) with a two-stream radiative transfer code to estimate surface irradiance. Deneke et al. (2008) used cloud retrievals from Meteosat in combination with RT simulations to estimate solar irradiance in the Netherlands. Mueller et al. (2009) used a lookup table approach for clouds with transmissions pre-calculated with RT and values interpolated from the lookup table. They used a cloud effective radius of 10 μm for water droplets using the Hu and Stamnes (1993) parametrisation of the phase function and did not consider ice clouds. While this may be sufficient for horizontal fluxes, this approach is less accurate when calculating the radiances required for the tilted irradiance. Behrendt et al. (2013) used the SOLIS clear-sky model with cloud adjustment to determine the spectral effects on different PV technologies. A separate run with clouds specified directly inside the radiative transfer model was performed. The difference in spectral transmission between SOLIS and the RT solution using the libRadtran package (Mayer and Kylling, 2005) is about 5% in average photon energy for thick cloud cover (optical depth of 60) at a solar zenith angle of 60°. More recently, the UniSky simulator software (Kocifaj and Fečko, 2014; Kocifaj, 2015) includes the effects of a 3D cloud field to model ground-level radiances. Current satellite products often include the required cloud optical properties, namely cloud phase (water or ice), cloud optical depth, and cloud droplet effective radius, to allow RT simulations including clouds to be performed. One motivation for inclusion of clouds inside the RT calculation is for the development of solar energy models that can be applied to a wide variety of historical, current and future datasets, for example meteorological reanalyses or climate models, as well as satellite observations. Another is the spectral effects of cloud attenuation are better captured with RT simulation, which is important for PV.

After the directional radiances have been calculated from the RT simulation, integrating the radiance field over the direction of interest will provide the tilted irradiance. McArthur and Hay (1981) used radiance distributions obtained from fish-eye photographs and obtained agreement to ±10% for horizontal diffuse irradiance and ±5% for tilted irradiance on a south-facing plane, in a variety of sky conditions. Brunger and Hooper (1993) derived an empirical model for the sky radiance distribution calculated from observations of clearness index (ratio of surface irradiance to extraterrestrial irradiance) and zenith angle. Similarly Gueymard (1987) derived the sky radiance distribution by producing different anisotropic sky radiance distributions for a clear-sky and an overcast sky. The all-sky radiance distribution was calculated as a weighted sum of the clear and overcast cases with cloud transmission as the weighting factor.

Other popular anisotropic tilted irradiance models (e.g. Bugler, 1977; Klucher, 1979; Willmott, 1982; Hay and Davies, 1980; Skartveit and Olseth, 1986; Reindl et al., 1990; Perez et al., 1990; Muneer, 1990) are varyingly complex functions of the horizontal diffuse and direct irradiance measurements along with solar position and panel orientation. A comparison between ten tilt models at the NREL site at Golden, Colorado, USA, found that most anisotropic models did not predict irradiance with a satisfactorily low error for tilted planes compared to the bounds of instrumental error from pyranometers (Gueymard, 2009). An intercomparison of 15 models (4 isotropic and 11 anisotropic) in Denmark, France and Spain again found that no one anisotropic model generally performed better than the others consistently when considering different cloud conditions, tilt angles and azimuth angles (Gracia-Amillo and Huld, 2013). Therefore, the continued development of tilt models for all-sky conditions is desirable.

In this paper, the optimal tilt angle of a fixed-angle solar collector is considered. For comparison with the PVGIS method, the panel is oriented towards the equator, although it is also possible to optimise azimuth as shown in Section 4.3. In the absence of horizon obstruction, shading, or radically different morning and afternoon weather conditions, the equatorial direction provides the best azimuthal alignment. The tilt angle of integration is varied to find the irradiance at each angle and summed over a year of operation to determine the optimal tilt. The model is tested against the tilted irradiance model in PVGIS and compared to tilted irradiance measurements from NREL.

2. Determining tilted irradiance from radiances

The irradiance on a tilted plane angled at tilt β and azimuth γ is a combination of the downwards and upwards radiance fields such that the bounds of the integration is over the hemisphere with base in the plane of the solar collector (Gueymard, 1987):

$$I_T = \int_0^{2\pi} \int_0^{\theta_m} L(\theta,\phi) \cos\theta_d \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi \tag{1}$$

where the angle between the normal of the tilted plane and the radiance direction of interest is given by

$$\cos\theta_d = \cos\beta\cos\theta + \sin\beta\sin\theta\cos(\phi-\gamma) \tag{2}$$

and the bound of the integration θ_m is in the plane of the solar collector such that

$$\theta_m = \frac{\pi}{2} - \tan^{-1}(\cos(\phi-\gamma)\tan\beta) \tag{3}$$

The radiance field L is calculated at a resolution of 3° in the polar direction and 10° in the azimuthal direction using the `DISORT` radiative transfer code (Stamnes et al., 2000), as part of the libRadtran package (Mayer and Kylling, 2005), with a pseudo-spherical correction to improve accuracy at low solar elevations (Dahlback and Stamnes, 1991). θ is the polar angle and ϕ is the azimuthal angle. The radiative transfer equation is solved numerically with 16 streams, the minimum recommended for calculating