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Porosity variation below a fluid–porous interface

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HIGHLIGHTS

- Derivation of a depth-dependent porosity relation below a fluid–porous interface.
- Assumption is based on spheres, randomly packed as a porous medium.
- Validation of relation by non-invasive laser method and numerical simulations.
- Comparison of the results with literature data on biofilm porosity.
- Showing effect of variable porosity on solute concentration profile in a biofilm.

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ABSTRACT

The correct quantification of porosity is essential in all studies pertaining to porous media. A host of existing works employs a constant, bulk value for porosity, even when the porous sample is attached to a free fluid. Since the volume fraction of the solid matrix near the interface region differs from that in the core, the porosity undergoes a spatial variation. Here we present a novel relation for the porosity as a function of depth below the interface, using the concept of surface roughness applied on the classical definition of open porosity. This relation has been verified by computational modeling as well as non-invasive laser experiments. It has been shown that this depth-dependent porosity relation applies also to a non-granular porous layer such as a biofilm.

1. Introduction

Porosity has been known to be the most significant property describing a porous medium. Moreover, its correct quantification is essential for flow, heat and mass transfer parameters such as permeability, tortuosity, thermal conductivity and diffusion coefficient.

In a variety of situations, a porous medium has a boundary with its non-porous surrounding such as a free fluid. Examples include microbial mats (Wieland et al., 2001), biofilms (Lewandowski, 2000), Rayleigh–Benard convection in fluid permeating a porous medium (Howle et al., 1993), monochromatic surface waves across fluid–porous interfaces (Albers, 2006), air-grain flows in granular media (Sandnes et al., 2010). In such cases, the porosity undergoes a spatial decrease due to increase of solid matrix density or packing density with depth. This fact has been already pointed out in previous studies (Ochoa-Tapia and Whitaker, 1995; Goyeau et al., 2003; Goharzadeh et al., 2005), however, a functional relationship for the depth-dependent porosity at fluid–porous interfaces has remained illusive.

Here we present a novel porosity–depth relation by applying the classical definition of open porosity to each thin slice of the porous layer below the fluid–porous interface. This relation is found to be applicable to granular as well as naturally growing porous layers.

To validate this relation, rigorous experimental and numerical investigations were performed. The experiments were composed of non-invasive planar laser induced fluorescence (PLIF) technique to visualize void and solid fractions below the interface. Solid matrix was considered to be mono-sized spherical glass beads.

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In the numerical approach, a free surface of randomly packed solid spheres was first generated. The porous layer underneath the free surface was then split into thin slices which were used to extract void and solid matrix fractions. Both experimental and numerical results agreed well with the porosity–depth relation presented.

Finally, the significance of inclusion of a depth-dependent porosity has been demonstrated on the example of oxygen concentration profile predicted for a biofilm.

2. Development of a new porosity–depth relation

We consider a cubic container of volume L^3 filled with a random packing of Nsmall solid spheres of diameter d having an interface with the overlaying fluid layer. Following previous studies (Beavers and Joseph, 1967; Neale and Nader, 1974; Goharzadeh et al., 2005) the position of the nominal fluid–porous interface, $y=0$, is defined as the location of the horizontal tangent to the perimeter of the uppermost sphere (Fig. 1). We introduce a transition layer with the thickness δ understood as the layer within which the porosity falls from unity at the fluid–porous interface to its bulk value in the porous core. To calculate the depth-dependent porosity, the transition layer is divided into an ensemble of M subsequent tiny horizontal slices of thickness, $\Delta = L/N$, occupied partially by fluid and partially by solid segments (top image in Fig. 1). The open porosity of each tiny horizontal slice can be given by

$$\varphi_m = 1 - \frac{1}{L^2 \Delta} \sum_{i=1}^{p} V_i \tag{1}$$

where V_i is the volume of each sphere segment and p is the total number of segments in a given slice m. The sum in Eq. (1) generally represents a trapezoidal volume similar to the so-called Tower of Hanoi (Buneman and Levy, 1980; Wolfram, 2011). It can be shown (see results section) that in a random packing of large number of spheres, the thickness Δ corresponds to the vertical distance which separates any two vertically closest spheres. With other words at a given depth there would exist only one sphere and the trapezoidal volume fades to the volume of a spherical cap $V(y)$ (right image in Fig. 1). Consequently, Eq. (1) converts to

$$\varphi(y) = 1 - \frac{N}{L^3} V(y) \tag{2}$$

with

$$V(y) = \pi d^3 \left[\frac{1}{2} \left(\frac{y}{d} \right)^2 - \frac{1}{3} \left(\frac{y}{d} \right)^3 \right]. \tag{3}$$

We recall that the volume fraction of solid spheres to the container volume can be given by

$$1 - \varphi_b = \frac{N \frac{\pi d^3}{6}}{L^3}. \tag{4}$$

Hence, upon substitution of Eq. (3) in Eq. (2) and elimination of N/L^3 from Eq. (2) using Eq. (4), one obtains the depth-dependent porosity relation as

$$\varphi(y^*) = 1 - (1 - \varphi_b) \left[3y^{*2} - 2y^{*3} \right]. \tag{5}$$

Here φ_b represents bulk porosity and $y^* = y/d$. The term $1-\varphi_b$ provides a universal constant which ranges from 0.609 to 0.641 for poured and close random packing (Dullien, 1992). The above relation has been derived under the premise that the solid spheres are randomly packed, provide point of contact with their neighboring ones, generate interconnected voids and are heavier than the saturating fluid.

3. Numerical and experimental methods

3.1. Numerical determination of porosity variation

To generate a random packing of N mono-sized spherical solid beads in a given cubic box of size L, a standard algorithm developed in Princeton University has been utilized (Skoge et al., 2006; Princeton, 2011). The Princeton code has been modified by us to construct a porous layer with a free interface with the fluid on top. To do so, we numerically remove the beads from the top of the box until we reach those beads with their centers below $y=0$ as shown in Fig. 2.

The constructed porous layer has the thickness $L/2+d/2$. Knowing the exact coordinates of the solid beads and voids, the porosity of each horizontal cross-sectional stripe of thickness Δ – representing the porosity at a given depth – is given by $\phi_i = A_{void} \cdot \Delta / A_{total} \cdot \Delta$. Hence, at any given depth the porosity can be extracted from the void to total area fraction. The repetition of this procedure for the entire whole porous layer leads to a numerically driven depth-dependent porosity relation.

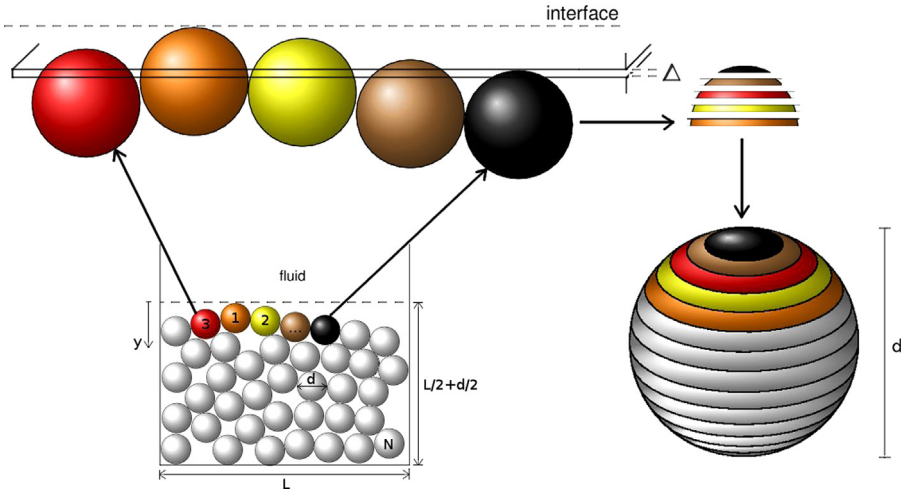

Fig. 1. A porous layer of solid spheres having a free interface with the upper fluid layer (bottom left image); any, sufficiently tiny, horizontal slice of thickness Δ (top image) will be occupied partially by solid and partially by void or fluid. Porosity at any depth is related to the volume of the segments confined within Δ constructing a spherical cap (bottom right image).