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Fast computation of solar concentrating ratio in presence of opto-mechanical errors

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Abstract

In this paper are defined analytic relationships allowing estimating the concentration ratio losses at the focus of any type of thermodynamic solar facilities affected with opto-mechanical errors. The approximate expressions are based on a simple Lorentz function, and are usable for introducing various families of defects, such as mirror shape accuracy, alignment and tracking errors, and astigmatism. These relations are not intended for replacing more sophisticated computing codes (Monte-Carlo or convolution models), but are suitable for first-guess performance prediction and preliminary system engineering and design.

Keywords: Concentration factor; Opto-mechanical errors; Focusing heliostat; Astigmatism

1. Introduction

In this paper are provided very simple analytic and approximate relationships for estimating concentration ratios achievable by solar thermodynamic systems in the presence of optical, mechanical and guiding errors. The analytical expressions seem to be applicable to most types of existing solar facilities, including focusing heliostats in the field of a central tower power plant, single-dish telescopes directly tracking the Sun, or double reflection solar furnaces. Moreover, they should be of particular interest when the solar facility is constituted of a large number of segmented surfaces or heliostats. Herein the simplified analytical relationship is demonstrated in Section 2. We discuss its applicability to various types of error sources in Section 3. The results of the model are compared with those obtained by using a ray-tracing software in Section 4, before giving a brief conclusion of this study in the last section.

2. Modelling the concentration ratio

We first define the theoretical "optical" concentration ratio C_0 of an optical system, excluding actual flux losses generated by the atmosphere, shadowing and blocking effects, or limited dimensions of the receiver aperture (i.e. interception efficiency):

$$C_0 = \frac{S \cos\theta}{S' \cos\gamma} = \frac{\Omega}{\Omega_0} \approx \frac{S \cos\theta}{\pi\varepsilon_0^2 D^2} = \frac{E'/\cos\gamma}{E_0}, \tag{1}$$

where all the employed physical parameters are described below.

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S The collecting area of the solar concentrating surface (usually mirrors) in m^2
θ The average incidence angle of the solar rays on the collecting surface in radians (the multiplying factor $\cos\theta$ is sometimes called "cosine effect")
S' The area of the receiving surface in m^2
γ The angle between the normal to the receiver and the impinging solar rays in radians (this actually represents a second cosine effect)
Ω The solid angle under which the collecting surface is seen from the receiver center in steradians
Ω_0 The solid angle of the Sun seen from ground level in steradians
ε_0 The angular radius of the Sun seen from ground level in radians
D The distance between the collecting and receiving surfaces in m
E' The irradiance generated at the center of the receiver in W/m^2
E_0 The standard solar irradiance at ground level in W/m^2, also named Direct normal irradiance (DNI)

We note that the first relationship on the left-hand side of Eq. (1) is rigorous, complying with the well-known conservation law of geometrical etendue. Conversely, the right-most relations are approximate, and are only applicable when the numerical aperture of the concentrating system is not too high, i.e. $S \ll D^2$, a very common case regarding focusing heliostats or sections of a parabolic concentrator illuminated by a flat heliostat, and to which this study will be limited. It can also be noticed that by definition the concentration ratio C_0 actually includes the first "cosine effect" $\cos\theta$, while the second one $\cos\gamma$ is left out.

Due to the opto-mechanical errors of the system and to the Sun's brightness shape, however, the irradiance E' at the receiver plane will generally not be uniform. This experimental fact will serve as starting point for the presented analytical model, that is based on the convolution formalism firstly described by Lipps (Lipps, 1976) and further generalized by Hénault and Bonduelle to different error sources (Hénault et al., 1987; Hénault and Bonduelle, 1989). Here the convolution product denoted by the symbol $\otimes$ is expressed in terms of dimensionless coordinates α and β normalized with respect to the nominal focal distance f of a given solar concentrating system. Alternatively, (α,β) could be considered as the angular coordinates of a fictitious sun shape integrating all the opto-mechanical errors of the concentrator, being projected back onto the sky. Then the irradiance distribution $E'(\alpha,\beta)$ formed onto the receiver plane can be written as:

$$E'(\alpha,\beta) = C_0 E_0 \cos\gamma \frac{[S\otimes P](\alpha,\beta)}{[S\otimes P](0,0)} \frac{[S\otimes P](0,0)}{S(0,0)},$$
$$\text{where}: [S\otimes P](\alpha,\beta) = \int_{u=-\infty}^{+\infty}\int_{v=-\infty}^{+\infty} S(u-\alpha, v-\beta)P(u,v)dudv. \quad (2)$$

Here $S(\alpha,\beta)$ and $P(\alpha,\beta)$ respectively stand for the Sun's angular brightness function and the probability density function of the angular errors affecting the whole concentrating system. Two different multiplying factors are appearing there, that can be understood as follows:

(1) A first term $[S\otimes P](\alpha,\beta)/[S\otimes P](0,0)$ stands for the spatial variations of the solar irradiance at the surface of the receiver. However the present paper mainly deals with the loss in concentrating ratio at the center of the receiver, where by definition $\alpha = \beta = 0$.
(2) The second term $[S\otimes P](0,0)/S(0,0)$ is directly characterizing the effects of opto-mechanical errors with respect to the ideal concentration ratio C_0 that can be achieved in the absence of any physical error, as illustrated in Fig. 1. Here the probability function $P(\alpha,\beta)$ in the denominator of Eq. (2) has simply been replaced with the Dirac's "Delta" distribution $\Delta(\alpha,\beta)$, whose integral is equal to unity.

Therefore from Eqs. (1), (2), the effective concentration C and the concentration loss factor η can be defined as:

$$C = \eta C_0 = C_0 \int_{u=-\infty}^{+\infty}\int_{v=-\infty}^{+\infty} S(u,v)P(u,v)dudv/S(0,0). \quad (3)$$

To derive a simple analytic formulation from Eq. (3), it is assumed that both functions $S(\alpha,\beta)$ and $P(\alpha,\beta)$ can be approximated by Gaussian laws, such that:

$$S(\alpha,\beta) = \exp\left[-(\alpha^2+\beta^2)/\varepsilon_0^2\right], \quad \text{and}: \quad (4a)$$

$$P(\alpha,\beta) = \frac{1}{\pi\delta^2}\exp[-(\alpha^2+\beta^2)/\delta^2], \quad (4b)$$

where ε_0 is the Sun radius at 1/e-height, and δ the quadratic sum of all considered opto-mechanical defects (see the following section). Combining Eqs. (2-4), and invoking some mathematical properties of convolution products allows to conclude that $E'(\alpha,\beta)$ shall be a normal distribution of standard deviation equal to $\sqrt{\varepsilon_0^2+\delta^2}$, and that the final

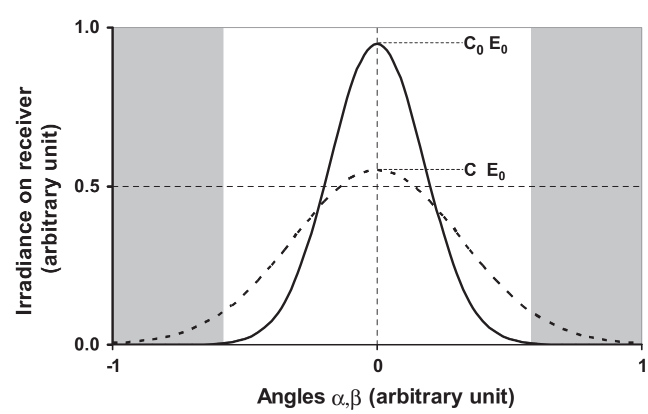

Fig. 1. Illustrating the concentrating ratio loss at the receiver center (sold line: maximal irradiance; dashed line: achieved irradiance). E_0 is the solar irradiance at ground level and C and C_0 are the concentration ratios (grey areas represent the flux loss outside of the receiver).