Chemical Engineering Science 105 (2014) 198–207

Contents lists available at ScienceDirect

Chemical Engineering Science

journal homepage: www.elsevier.com/locate/ces

Lifting a buried object: Reverse hopper theory

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HIGHLIGHTS

- Formulae predict force to push up an object buried in granular material.
- The formulae are for (1) axi-symmetric motion and (2) two-dimensional motion.
- Axi-symmetric motion implies granule movement within a cone, reverse hopper flow.
- Two-dimensional motion implies granule movement within a triangular prism.
- The theoretical push-up forces agree well with published experimental data.

ARTICLE INFO

Article history:
Received 9 July 2013
Received in revised form
29 October 2013
Accepted 2 November 2013
Available online 11 November 2013

Keywords:
Mathematical modelling
Fluidisation
Granular materials
Particle
Buried object
Lifting

ABSTRACT

A theory is given to predict the upward force, F, to lift an object buried at depth H in non-cohesive granular material. Explicit formulae give F in terms of the material friction coefficient of the granular material and the geometric parameters.

The lifted object is either (1) a horizontal disc of diameter D or (2) a horizontal plate of width B and length L, where $L \gg B$. In case (1), the lifted disc is assumed to cause axi-symmetric upward particle motion, *reverse hopper flow*, within an inverted cone. *Active* failure is assumed: the vertical stress, σ_2, is $K \times$ (horizontal stress σ_1); here $K = (1+\sin\phi)/(1-\sin\phi)$, ϕ being the angle of friction for the granular material. This gives the vertical stress, σ_{20}, on the disc. An additional lift force is needed to overcome the frictional stress, τ, at the conical interface between stationary and upward moving particles: it is assumed that $\tau = \mu\sigma_1$, μ being the internal friction coefficient. For consolidated granules, $\mu = \tan\phi$, but for the sheared material, $\mu < \tan\phi$. The total lift force F is the sum of (i) the effect of σ_{20} plus (ii) the effect of τ; this sum gives an equation to predict the *breakout factor* $N_{qf} = F/(\gamma' AH)$, where γ' = bulk weight density and $A = \pi D^2/4$. For case (2), relevant to the uplift of a long buried pipe, the theory is similar: the two failure surfaces are flat, inclined at angles $+\alpha$ and $-\alpha$ to the vertical. Similar assumptions as to the stress distribution, i.e. two-dimensional *active* failure, give an equation for N_{qf}. The two predictive equations for cases (1) and (2) agree well with relevant published measurements of N_{qf}.

1. Introduction

This work began with a study of gas distributors for industrial fluidised beds. Such distributors often consist of a flat horizontal plate containing a number of nozzles through which gas flows to fluidise the particles above. The nozzles are separated by a horizontal distance, typically 10–20 cm; thus the bed of particles is fluidised by upward facing gas jets emerging from nozzles which may be a few centimetres diameter, separated by the horizontal distance 10–20 cm mentioned above.

Particles within each nozzle need only a modest gas flow for fluidisation, far less than the flow required to fluidise the main particle bed supported on the distributor. The question then arises: '*At what gas velocity through each nozzle will the whole bed be fully fluidised and what is the behaviour of the particles immediately above each nozzle*?' To fluidise the whole bed, a superficial velocity U_{mf} is needed, when the velocity in each nozzle will be $A_b U_{mf}/a$ where A_b is the bed area and a is the total area of the nozzles. Particles in the nozzles will be more than fully fluidised, because A_b/a is large. This paper deals with the situation where the nozzle gas velocity, U_N, is intermediate, i.e. $U_{mf} < U_N < A_b U_{mf}/a$. Thus each nozzle is fully fluidised but the main bed is not. In this situation, the particles above the nozzle will be subjected to forces of two kinds namely

(i) pressure gradients in the gas due to the outflow of gas from each nozzle into the main bed, and
(ii) interparticle forces arising from the particles near the nozzle having an interstitial gas flow in excess of U_{mf}: this gas flow

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0009-2509/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ces.2013.11.002

pushes the particles upwards so that they can press on the particles above, resulting in interparticle stresses.

This paper is concerned with these interparticle stresses and attempts to answer the question: '*What is the level of interparticle stress needed to push up the particles above*?' This question is similar to the question posed by workers in soil mechanics: '*If an object is buried in granular material, what upward force is needed to lift it*?' The answer to the latter question is given here by using well-established theory of hopper flow, but in reverse, i.e. upward rather than downward particle motion. In hopper theory (Savage, 1965; Sullivan, 1973; Davidson and Nedderman, 1973; Savage and Sayed, 1981) granular material flows out of a hopper, as in an hour glass. The theory, based on the assumption of frictionless hopper walls, gives values which agree qualitatively with experimental data, e.g. that the flow rate is proportional to (outlet diameter)$^{5/2}$. For hopper flow, the granular material is in the *passive state*, i.e. (horizontal stress)$=K\times$(vertical stress), where $K=(1+\sin\phi)/(1-\sin\phi)$, $\tan\phi$ being the internal friction coefficient of the granular material.

In reverse hopper flow, the pushed up material is assumed to be in the *active state*, i.e. (vertical stress)$=K\times$(horizontal stress). There is the further assumption that the displaced material moves within a cone whose axis is vertical. With these assumptions, the force at the bottom of the cone can be calculated. This force due to the normal stresses is less than the observed force. This appears to be because the conical interface between the upward moving and stationary material is assumed to be smooth. When a frictional force is allowed for, the total uplift force appears to be in good agreement with a wide range of experimental results.

The *purpose of this paper* is to present the newly developed reverse hopper theory for predicting the uplift force against a surcharge of granular material. Two versions of the theory are given:

(a) *Axi-symmetric*: Here the pushed up granular material is assumed to be of conical form, i.e. the motion is axi-symmetric.
(b) *Two-dimensional*: Here the boundaries between upward moving granules and stationary granular material are assumed to be two flat interfaces symmetrically aligned about the central vertical axis, forming a two-dimensional flow pattern.

2. Literature review

The published research into fluidisation is primarily focused on predicting the hydrodynamics of fluidised systems with relatively few published works related to the start-up of industrial fluidised beds. Besides the well known group classification of Geldart (1973), which enables the prediction of some fluidisation properties in terms of particle size and density, the minimum fluidising velocity and the bed expansion characteristics (as a function of bed porosity) are the two most widely studied design parameters for fluidised beds. Di Felice (1995) presented a comprehensive review on the theoretical development regarding the hydrodynamics of liquid fluidisation.

A similar problem to that investigated in this work is the uplift of anchors and pipes, studied in geotechnical engineering. The following section reviews the relevant research in geotechnical engineering.

2.1. Insights from geotechnical problems

There are two main types of uplift problem in geotechnical engineering. The first category is related to the upheaval buckling of offshore pipelines buried in the seabed. These pipelines transport high-temperature oil and undergo upheaval buckling as a result of thermal expansion (Ireland and Bransby, 2009). This phenomenon is one of the key failure modes of such pipelines and has serious economic and environmental consequences (Cheuk et al., 2008).

The second category is related to the study of plate anchors buried in sand. These plates are used in foundation systems where they are fixed to a building structure and embedded in the ground at sufficient depth to resist pullout forces (Merifield and Sloan, 2006).

For both applications, the uplift force is important. Thus the main design challenge for buried plates or pipelines is to determine the minimum depth of soil cover that will provide sufficient uplift resistance. The uplift resistance provided by the soil cover increases with depth, but the cost of the burial increases with depth and constitutes a significant fraction of the total construction cost. Therefore, burial depth should be minimised while providing sufficient uplift resistance (Cheuk et al., 2008). This is the motivation behind most of the work in geotechnical engineering.

Currently, there are two main approaches to model the uplift of anchors and pipes in soils. The first approach uses variations of the Vertical Slip Model of Matyas and Davis (1983a). This is often referred to as the limit equilibrium method. This assumes that the uplift force is the sum of the weight of the soil being lifted and the shear force along the failure surface. Recently, White et al. (2008) presented a limit equilibrium solution for the vertical pullout of pipes and plate anchors buried in sand. The authors included the stress-dilatancy correlations presented by Bolton (1986) in their equilibrium solution. The second approach is the finite element method, successfully applied by Merifield and Sloan (2006) to predict the deformation mechanism during anchor uplift.

The uplift experiments reported in the literature were mostly carried out in a laboratory scale tank containing sand. The plates or pipes were embedded in the sand at a specified depth and connected, via a rod, to a load cell fixed above the tank. They were displaced at a constant pull-out rate using a gearbox. The displacement and uplift force were continuously monitored and recorded using displacement transducers and load cells.

A three-region behaviour, illustrated in Fig. 1, is usually observed in uplift tests reported in the literature (Cheuk et al., 2008; Ilamparuthi et al., 2002; Murray and Geddes, 1987; Palmer et al., 2003).

Referring to Fig. 1, the first region is the pre-peak behaviour, exhibiting a rapid increase in load. The second region is the post-peak behaviour; the load decreases rapidly as displacement increases. The third region is the residual behaviour, associated with a gradual decrease in load at large displacements (Ilamparuthi et al., 2002).

Cheuk et al. (2008) presented a detailed analysis on the three-region behaviour illustrated in Fig. 1. Fig. 2 summarises the results of Cheuk et al. (2008) on the deformation mechanism during pipe uplift using PIV (particle image velocimetry). Fig. 2(a) shows the

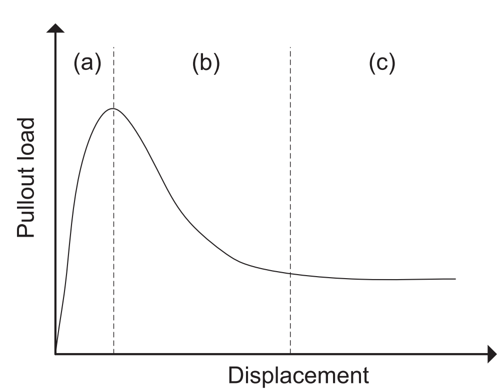

Fig. 1. Typical relationship between pullout load and displacement for uplift of plates/pipes in sand, with three different regions indicated in the figure.