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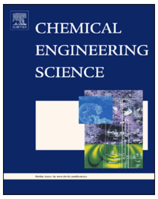

Three-dimensional tool design for steady-state electrochemical machining by continuous adjoint-based shape optimization

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HIGHLIGHTS

- We propose a method to solve the tool design problem in electrochemical machining in two and three dimensions.
- We use continuous adjoint-based shape optimization combined with elements of shape calculus.
- A comparison with exact analytic solution demonstrates the accuracy of the method.
- We investigate the influence of electrode curvature on the front gap width.
- The inaccuracies of the cos θ-method for curved electrodes can be estimated.

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ABSTRACT

In electrochemical machining (ECM), it is important to design the shape of an appropriate tool capable of producing a workpiece of desired shape. This work presents a numerical approach to solving the two- and three-dimensional tool design problem in steady-state ECM. The tool design problem is transformed into a shape optimization problem and then solved using the continuous adjoint method combined with elements of shape calculus, ensuring high efficiency and the maximum possible degrees of freedom. A numerical experiment on a two-dimensional Gaussian-shaped workpiece shows a good agreement of the calculated tool shape with the exact analytical solution. Tool design is carried out for a series of two- and three-dimensional workpiece shapes to investigate the influence of the curvature of the desired workpiece on the front gap width in steady-state ECM.

1. Introduction

Electrochemical machining (ECM) is a non-conventional materials processing technique used to produce parts of complex shapes made of high-strength alloys, irrespective of their hardness and without introducing residual stress. For the last five decades, ECM-related techniques have been developed intensively. Already to the end of 1974, more than 1200 related scientific articles were published (Bannard, 1977); several review articles (Bannard, 1977; Rajurkar et al., 1999) and monographs (De Barr and Oliver, 1968; Faust, 1971; McGeough, 1974) provide an overview of the topic. Today, applications of ECM range from the production of turbine disks and blades for aerospace industry (e.g. Lamphere et al., 2007) to shaping three-dimensional (3D) structures with submicrometer precision (e.g. Schuster et al., 2000).

In an ECM process depicted schematically in Fig. 1(a), a metallic tool of defined shape is advanced towards a metallic workpiece at a specified tool feed rate (velocity). The interelectrode gap is filled with appropriate electrolyte (e.g. NaCl, $NaNO_3$). An external DC voltage is applied such that the negatively polarized tool is the cathode, and the positively polarized workpiece is the anode of an electrolytic cell.[1] Electric current flows across the interelectrode gap due to electrochemical reactions at the surfaces of both electrodes. While the tool shape is not changed since hydrogen or nitrate (Mao, 1971) reduction usually takes place at its surface, metal atoms at the workpiece are oxidized to ions. These ions dissolve into the electrolyte which is pumped through the interelectrode gap to prevent boiling by Ohmic heating and to sweep away the reaction products. Due to the current density distribution at its surface, the shape of the workpiece approaches the negative image of the tool, as schematically shown in Fig. 1(b).

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[1] In this work, the expressions tool/cathode and workpiece/anode are used as “synonym pairs”, respectively.

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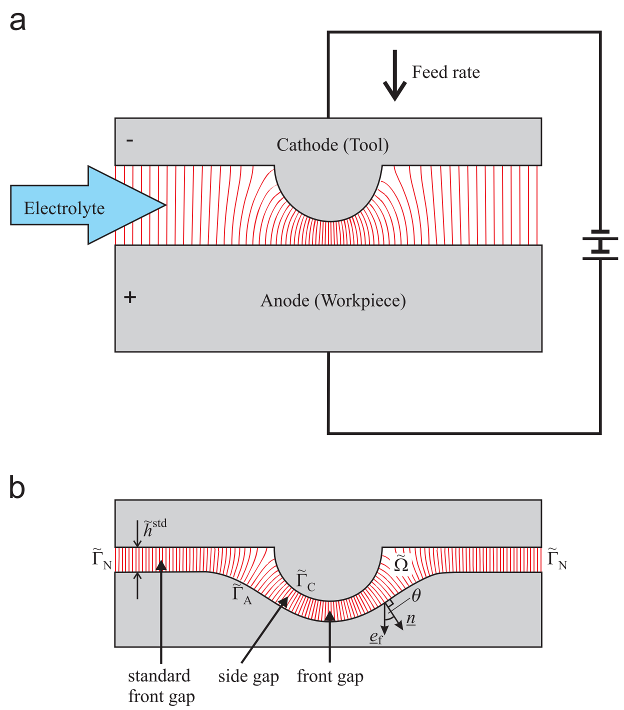

Fig. 1. Basic principle of electrochemical machining (ECM). Panel (a) shows the initial machining configuration consisting of a cathode tool of a defined shape and a plane anode workpiece to be machined. The tool is advanced towards the workpiece at a defined feed rate. Electric current flows when a voltage is applied across the gap; the thin red lines between the electrodes illustrate the current density. Panel (b) shows the steady state configuration of tool and workpiece. The final shape of the workpiece is approximately the negative image of the tool electrode. The inter-electrode gap is the calculation domain $\tilde{\Omega}$ of the model used in this work. The boundaries $\tilde{\Gamma}_{\mathrm{C}}$ and $\tilde{\Gamma}_{\mathrm{A}}$ are defined by tool (cathode) and workpiece (anode) surfaces, respectively, and $\tilde{\Gamma}_{\mathrm{N}}$ is the side boundary through which the electric current flux is zero. For each point on the workpiece boundary $\tilde{\Gamma}_{\mathrm{A}}$, θ is defined as the angle between the outer boundary unit normal vector $\underline{n}$ and the feed direction unit vector $\underline{e}_{\mathrm{f}}$. “Front gap” refers to an inter-electrode gap region where the workpiece surface is perpendicular to the feed direction (i.e. $\theta = 0$). “Standard front gap” with width $\tilde{h}^{\mathrm{std}}$ is a front gap region where the electrodes are plane. All other regions of the inter-electrode gap are called “side gap”. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

Apart from understanding the physical phenomena related to high-rate metal dissolution (e.g. Datta, 1993) and realizing reliable process control, a further problem associated with ECM concerns accurately relating the shape of the workpiece to that of the tool. The *direct problem* in ECM refers to predicting the workpiece shape evolution for a given tool shape, whereas the *inverse problem* deals with designing the shape of the tool capable of producing a desired workpiece shape under specified machining conditions.

The direct problem in ECM is related to the cathode shape change problem in electrodeposition; both have been studied using analytical and numerical techniques. Fitz-Gerald and McGeough (1969) expanded two-dimensional (2D) electrode shapes into Fourier series to analyze surface smoothing during ECM analytically. Many numerical simulations were based on the so-called potential model, where the primary current distribution (definition cf. Ibl, 1981) is taken into account. Alkire et al. (1978) employed the finite element method (FEM) to predict the cathode shape change in electrodeposition. Sautebin et al. (1980) used FEM to calculate workpiece surface smoothening in ECM. Białecki et al. (1984) and Deconinck (1994) used the boundary element method (BEM) to calculate primary current distribution in ECM. Concepts of 3D ECM computer simulations were proposed by Davydov et al. (2004); practical 3D ECM simulations based on the primary current distribution were implemented in commercial software packages (e.g. Purcar et al., 2004).

Beyond the potential model, Prentice and Tobias (1982) calculated changing electrode profiles in electrodeposition, taking electrode kinetics into account by boundary conditions based on the Butler–Volmer equation. Hourng and Chang (1993) and Kozak (1998) developed mathematical models for ECM-related process simulations accounting for the influence of Ohmic heating and gas fraction on the conductivity. Electrolyte flow in a 3D ECM model was considered by Fujisawa et al. (2008). Recently, complex ECM simulations involving multi-ion transport were presented (Deconinck et al., 2012a).

The inverse tool design problem, which is the topic of this work, is more important for industrial applications. To our knowledge, main non-experimental approaches to this problem encompass approximative analytical, exact analytical, iterative numerical, and non-iterative numerical methods.

The so-called $\cos\theta-$method is an approximative analytical method employed by Tipton (1971), assuming the gap width to be inversely proportional to $\cos\theta$, where θ is the angle between the tool feed direction and the normal of the workpiece boundary (cf. Fig. 1(b)). According to De Barr and Oliver (1968), this method is applicable for $\theta < 60^\circ$. But even for the front gap ($\theta = 0$), the $\cos\theta-$method will only yield results with inaccuracies of less than 10% if the radius of curvature of the workpiece surface is at least one order of magnitude larger than the gap width, as will be shown in Section 4.

The exact analytical method for 2D tool design problems proposed by Krylov (1968), Nilson and Tsuei (1974) relies on the harmonic property of real- and imaginary parts of holomorphic functions and the principle of analytic continuation. For workpiece shapes that are representable by $y = F(x)$ with an analytic function F, the shapes of the tool can be obtained as parametric curves in 2D. Alder et al. (2000) combined this method with Fourier series to represent more general 2D workpiece shapes. For another exact approach proposed by Lacey (1985), a harmonic function Ψ has to be determined in a way that its gradient describes the position vector field of the workpiece surface. The method is exemplarily applied to a special 3D problem, but calculation of Ψ for general workpiece shapes is difficult.

A numerical approach to the ECM tool design problem without tool movement is reformulating it as an elliptic variational inequality (Elliott, 1980). Butt (1993) presented applications of this method to the dissolution of a workpiece that is fully surrounded by the tool. Liu and Rubio (1991) proved the existence of the solution mathematically. Although the industrially more relevant case with moving tool electrode was not presented, the method could be used for tool design in non-steady-state ECM (Butt, 1993).

Starting from an initial guess of the tool shape, some researchers correct the tool iteratively based on numerical solutions of the direct problem to overcome the ill-posedness of the inverse problem. Reddy et al. (1988) developed a correction factor method that adjusts the tool shape based on the calculated anode profile to an assumed tool. Narayanan et al. (1986) and Hardisty and Mileham (1999) converted the deviation of the current density on a workpiece point from the desired value into a positional error of the tool point along the same current flux line, which is then used to correct the tool. Hunt (1990) employed an embedding method that searches among the solutions of the direct problem.

Iterative numerical tool design methods can also be based on optimization. Das and Mitra (1992) used BEM, Zhou and Derby (1995) used FEM to calculate the electric potential and defined the sum of squared difference between actual and required current density at all node points at the workpiece boundary as the objective function to be minimized. The tool is represented by Fourier series with coefficients as design variables. In the optimization procedure, partial derivatives of the objective function with