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Order reduction and control of hyperbolic, countercurrent distributed parameter systems using method of characteristics

Sudhakar Munusamy [a], Sridharakumar Narasimhan [a,*], Niket S. Kaisare [b]

[a] Department of Chemical Engineering, Indian Institute of Technology Madras, Chennai 600036, India
[b] ABB Corporate Research Center, Bhoruka Tech Park, Whitefield Main Road, Bangalore 560048, India

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ABSTRACT

The aim of this paper is to develop a model reduction technique based on method of characteristics (MOC) for control of counter-current distributed parameter systems that are modeled by semi-linear hyperbolic partial differential equations (PDEs). In our previous work, MOC was shown to be a suitable model reduction technique for a class of hyperbolic PDEs. This concept is extended to counter-current systems, wherein the so-called characteristic lines have slopes with opposite signs. Two different approximations are proposed that allow the use of MOC as a model reduction technique. The open-loop results from MOC are compared with a large dimensional model, based on the method of lines. The MOC-based models are used for closed loop simulations within the approximate dynamic programming (ADP) framework. Two case studies are considered: a non-adiabatic plug flow reactor (having characteristics with two different slopes) and a non-adiabatic fixed bed reactor (having characteristics with three different slopes). We demonstrate that using the MOC-based model in an ADP controller results in a significant improvement in computational time, along with a slight improvement in controller performance.

1. Introduction

Convection dominated distributed parameter systems are often encountered in process industry. Examples for such systems include plug flow reactors, fixed bed reactors, heat exchangers, etc. These systems are described by first order hyperbolic partial differential equations (PDEs). Control of such systems typically requires design of infinite dimensional controllers whose design and implementation are complicated (Balas, 1986; Lo, 1973; Choe and Chang, 1998; Christofides and Daoutidis, 1998, 1996). For example, Christofides and Daoutidis (1998, 1996) have designed a controller using output-feedback methodology for these systems. Here the designed control input is distributed in space and time and the design is based on the feedback linearized model. Traditionally, design of model-based controllers employ PDE based models which often use finite-dimensional approximations using finite difference or finite element methods. However, these approximations still result in large-dimensional models, which may not be well-suited for model-based control (Midhun and Kaisare, 2011; Dochain et al., 1992; Sorensen et al., 1980). The difficulty in obtaining reduced dimensional approximations for first order hyperbolic PDEs was clearly mentioned by Dubljevic et al. (2005).

This motivates the need for model reduction techniques to obtain reduced order models. Proper orthogonal decomposition is one of the popular order reduction techniques and is based on the modal decomposition which produces low order models by discarding modes with low energy (Padhi and Balakrishnan, 2003; Shvartsman and Kevrekidis, 1998; Pitchaiah and Armaou, 2010; Shvartsman et al., 2000). This method is not suitable for hyperbolic PDEs, as these equations exhibit modes with nearly equal energy. Thus the use of finite difference or POD based order reduction method for hyperbolic PDEs results in artificial diffusion which is not physical. Hence order reduction of first order hyperbolic equations needs special attention. The method of characteristics (MOC) is a powerful solution technique applicable to hyperbolic PDEs (Hanczyc and Palazoglu, 1995; Shang et al., 2004; Mohammadi et al., 2010; Fuxman et al., 2007; Shang et al., 2007; Choi, 2007; Choi and Lee, 2004, 2005). In our previous work (Sudhakar et al., 2013) we have demonstrated the use of method of characteristics in obtaining a reduced order model for a class of systems described by hyperbolic PDEs. The MOC-based reduced order model may then be used in model-based control such as model predictive control (MPC). We demonstrated the use of MOC based reduced order model in MPC and approximate dynamic programming (ADP) (Sudhakar et al., 2013).

In this work, we focus on using MOC to obtain control-relevant reduced order models for counter-current systems. The state variables vary along certain 'characteristic lines', which are

* Corresponding author. Tel.: +91 44 22574177; fax: +91 44 22574152.
E-mail address: sridharkrn@iitm.ac.in (S. Narasimhan).

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determined by the coefficient of convective term. Physically, one can visualize this as 'the flow of information' occurring in the direction of convective flow in the system, in the absence of diffusive term. Consequently, systems with single flow and co-current systems have characteristic lines with only positive slopes (Sudhakar et al., 2013). Unlike such systems, counter-current systems have characteristic lines with both positive and negative slopes. Additionally, two different approximations are suggested, resulting in two different implementations of MOC to obtain the reduced-order model.

ADP has generated a lot of interest in the past decade due to its potential for giving an optimal closed-loop performance at significantly lower online computational cost (Kaisare et al., 2003; Tosukhowong and Lee, 2009; Padhi and Balakrishnan, 2003; Midhun and Kaisare, 2011; Lee et al., 2006). The use of reduced order models in ADP reduces the problem of 'curse of dimensionality', the major issue in this technique. Further, it results in improved closed loop performance with reduced online computational load. Thus in the present work, the ADP based controller is designed using the reduced order model obtained from the application of method of characteristics.

Two different case studies of counter-current reactors are discussed: one containing characteristic lines with two different slopes, the second with three different slopes. The latter case, involving characteristics with three slopes, is more complex. Thus in the present work we have proposed a novel methodology to obtain a reduced order model for the counter-current system described by first order hyperbolic PDEs. The use of this reduced order model is demonstrated in the closed loop simulation using approximate dynamic programming.

The organization of this paper is as follows: Initially MOC as an order reduction method is introduced and the methodology of obtaining reduced order models is explained in detail. Next, the theory behind ADP and the advantages of using reduced dimensional models in ADP are explained. This is followed by presentation of the simulation results using MOC and ADP for the two case studies involving characteristics lines with two and three slopes. The overall advantages and shortcomings in employing reduced order models from MOC in ADP based control are then summarized.

2. Reduced order model using the method of characteristics

MOC is a technique for solving first order hyperbolic PDEs. It is based on finding a relation between two independent variables,

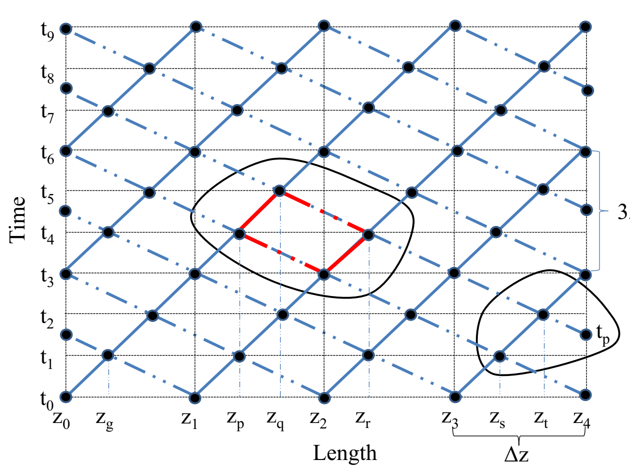

Fig. 1. Schematic figure showing intersection of characteristic lines in the time–space plane.

time and space, through certain lines called characteristic lines. This results in reducing the PDEs to family of ODEs along these characteristic lines in the time–space plane. Since the ODE representation along the characteristic lines is exact, MOC produces more accurate results than any finite discretization schemes with the same number of nodes. The accuracy of the MOC solution depends on the number of characteristic lines used to approximate the solution surface.

This work deals with counter-current systems involving characteristic lines with positive and negative slopes. The state variables vary along these characteristic lines. Solutions are obtained at points of intersection of the characteristic lines (also called 'nodes') in the time–space plane. The ratio of the slopes determines the location of such intersection points.

2.1. PDEs having characteristic lines with opposing slopes

A general convection-dominated counter-current system may be represented using the following semi-linear hyperbolic PDEs:

$$\frac{\partial \Phi_1}{\partial t} = -\psi_1 \frac{\partial \Phi_1}{\partial z} + f_1(\Phi_1, \Phi_2) \tag{1}$$

$$\frac{\partial \Phi_2}{\partial t} = \psi_2 \frac{\partial \Phi_2}{\partial z} + f_2(\Phi_1, \Phi_2). \tag{2}$$

The boundary and initial conditions are given by

$$\begin{bmatrix} \Phi_1(0,t) \\ \Phi_2(L,t) \\ \Phi_1(z,0) \\ \Phi_2(z,0) \end{bmatrix} = \begin{bmatrix} \Phi_{1,in} \\ \Phi_{2,in} \\ \Phi_{1,0}(z) \\ \Phi_{2,0}(z) \end{bmatrix} \tag{3}$$

Here $\Phi_1 \in \Re^{n_1}$, $\Phi_2 \in \Re^{n_2}$, $\psi_1, \psi_2 \in \Re^+$. f_1 and f_2 are nonlinear functions of appropriate sizes. Let $z_a(t; z^0, t^0)$ and $z_b(t; z^0, t^0)$ represent the characteristic lines with two distinct slopes starting from (z^0, t^0). The equation for such characteristics lines as derived from Eqs. (1) and (2) is given by Knuppel et al. (2010) and Sudhakar et al. (2013),

$$\begin{bmatrix} \frac{dz_a}{dt}(t; z^0, t^0) \\ \frac{dz_b}{dt}(t; z^0, t^0) \end{bmatrix} = \begin{bmatrix} \psi_1 \\ -\psi_2 \end{bmatrix} \tag{4}$$

The equation for the dependent variables along these characteristic lines as derived from Eqs. (1) and (2) is given by Knuppel et al. (2010), Sudhakar et al. (2013),

$$\begin{bmatrix} \frac{d\Phi_1}{dt}(t, z_a(t; z^0, t^0)) \\ \frac{d\Phi_2}{dt}(t, z_b(t; z^0, t^0)) \end{bmatrix} = \begin{bmatrix} f_1(\Phi_1, \Phi_2) \\ f_2(\Phi_1, \Phi_2) \end{bmatrix} \tag{5}$$

Fig. 1 is a schematic figure showing the characteristic lines with two slopes having opposite signs. The points of intersection of the characteristic lines show a periodic pattern (Mohammadi et al., 2010) which depends, in general on the ratio ψ_1/ψ_2. The starting points for the characteristic lines (z^0, t^0) are depicted as '•' in Fig. 1. The desired order reduction determines the space interval Δz at which the characteristic lines are placed and the time interval Δt is determined by the intersection of the two characteristic lines. For example, the characteristic lines $z_a(t; z_0, t_0)$ and $z_b(t; z_1, t_0)$ intersect at (z_g, t_1) and thus $\Delta t = t_1 - t_0$. For the purpose of illustration Fig. 1 is shown with 5 nodal points (z_0, z_1, z_2, z_3 and z_4) and the periodic pattern is shown as $3\Delta t$.

Eq. (5) has to be solved along these characteristic lines, starting at a node (z^0, t^0) and the solution is obtained at the intersection points. In order to solve Eq. (5), one needs the simultaneous variation of both the variables (Φ_1 and Φ_2) as required by the functions f_1 and f_2 along any characteristic line. But starting from any point, each equation in Eq. (5) is solved along different