Available online at www.sciencedirect.com

SciVerse ScienceDirect

Solar Energy 86 (2012) 1416–1427

SOLAR ENERGY

www.elsevier.com/locate/solener

An analytical expression for the instantaneous efficiency of a flat plate solar water heater and the influence of absorber plate absorptance and emittance

D.E. Roberts [a,*], A. Forbes [a,b]

[a] Council for Scientific and Industrial Research, National Laser Centre, Building 46, P.O. Box 395, Pretoria 0001, Gauteng, South Africa
[b] School of Physics, University of KwaZulu-Natal, Private Bag X54001, Durban 4000, South Africa

Received 22 September 2011; received in revised form 19 January 2012; accepted 31 January 2012
Available online 11 March 2012

Communicated by: Associate Editor H.-M. Henning

Abstract

Standard test results to quantify the instantaneous efficiency, η, of a glazed flat plate solar water heater are normally expressed in terms of a reduced temperature parameter, x, and global insolation, G, as $\eta = \eta_0 - a_1x - a_2Gx^2$. We show that the Hottel–Whillier–Bliss relation for the efficiency can be expressed in the same form with each of the coefficients η_0, a_1, and a_2 in terms of algebraic expressions of standard mechanical, fluid and thermal parameters of a single glazed, finned heater, including the absorber plate absorptance, α, and thermal emittance, ε. The advantage of the derived expression is that the effect on the efficiency of changes in various heater parameters can be readily evaluated. Furthermore, it is shown that for selectivity $\alpha/\varepsilon > 2$, each coefficient η_0, a_1, and a_2 can be expressed as $\eta_0 = \eta_{0C} - \varepsilon\eta_{0R}$, etc., in order to separate out the role of absorber radiation from other losses. This allows one to easily compare selective solar absorbers with different α and ε and, for example, to suggest an optimum coating thickness for thickness sensitive selective solar absorbers. In particular it can be seen that care should be taken in reducing ε at the expense of also reducing α in order to increase the selectivity, α/ε, since this will often be detrimental to the efficiency. The analytical expressions for η_0, a_1, and a_2 can be easily programmed on a spreadsheet and, for convenience, are summarised in an appendix.

Keywords: Flat plate solar heater; Efficiency; Analytical model; Selective absorber

1. Introduction

A large percentage of the solar heating market is for the production of hot water for domestic consumption, while a large fraction of this segment still relies on flat plate solar heaters (Norton, 2011). An important part of the testing of such heaters under operational conditions according to international procedures (ASHRAE 93, 2003; EN 12975-2, 2006; ISO 9806-1, 1996; Rojas et al., 2008) is the measurement of the instantaneous efficiency, η, as a function of the reduced temperature parameter:

$$x = \frac{T_m - T_a}{G} \tag{1}$$

The measured results are compared with the quadratic relation:

$$\eta = \eta_0 - a_1x - a_2Gx^2 \tag{2}$$

The best fit values of η_0, a_1 and a_2 are then quoted as measures of the heater efficiency. A heater should ideally have η_0 as large as possible and a_1 and a_2 as small as possible so that hot water can be supplied over a wide range of solar insolation.

* Corresponding author. Tel.: +27 12 841 4851; fax: +27 12 841 3152.
E-mail address: troberts@csir.co.za (D.E. Roberts).

0038-092X/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.solener.2012.01.032

Nomenclature

η	instantaneous collector efficiency
x	reduced temperature parameter (°C W^{-1} m^2) or (KW^{-1} m^2)
η_0	collector efficiency for $x = 0$
a_1	linear in x efficiency parameter (Wm^{-2} K^{-1})
a_2	quadratic in x efficiency parameter Wm^{-2} K^{-2})
G	incident solar power (Wm^{-2})
α	absorber solar absorptance
ε	absorber thermal emittance
a	absorber absorption coefficient
λ	wavelength (m)
ε_a	ambient thermal emittance
ε_p	cover plate thermal emittance
τ	cover plate transmission
F_R	collector heat removal factor
F'	collector efficiency factor
F	fin efficiency
U_L	total heat loss coefficient (Wm^{-2} K^{-1})
U_{Lt}	top heat loss coefficient (Wm^{-2} K^{-1})
U_{Lb}	bottom heat loss coefficient (Wm^{-2} K^{-1})
A	area of collector (m^2)
L	flow tube length (m)
k_c	thermal conductivity of collector ($Wm^{-1}K^{-1}$)
δ	collector absorber plate thickness (m)
m	$\sqrt{(U_L/(k_c\ \delta))}$ (m^{-1})
D	diameter of tube (m)
W	distance between fins (m)
σ	Stefan–Bolzmann constant (5.6704×10^{-8} Wm^{-2} K^{-4})
T_a	ambient temperature (°C)
T_c	average collector plate temperature (°C)
T_m	mean fluid temperature (°C)
T_{in}	inlet water temperature (°C)
T_{out}	outlet water temperature (°C)
h_{fi}	heat transfer coefficient fin to fluid (Wm^{-2} K^{-1})
h_{cca}	total convective heat loss coefficient (collector absorber to ambient) (Wm^{-2} K^{-1})
h_{ccp}	total convective heat loss coefficient (collector absorber to plate) (Wm^{-2} K^{-1})
h_{cpa}	total convective heat loss coefficient (plate to ambient) (Wm^{-2} K^{-1})
h_{rca}	radiative heat loss coefficient (collector absorber to ambient) (Wm^{-2} K^{-1})
h_{rcp}	radiative heat loss coefficient (collector absorber to plate) ($Wm^{-2}K^{-1}$)
h_{rpa}	radiative heat loss coefficient (plate to ambient) (Wm^{-2} K^{-1})
h_0	temperature independent convection heat loss coefficient, absorber to cover plate (Wm^{-2} K^{-1})
h_1	temperature dependent convection heat loss coefficient, absorber to cover plate (Wm^{-2} K^{-2})
h_2	wind speed independent convection heat loss coefficient, cover plate to ambient (Wm^{-2} K^{-1})
h_3	wind speed dependent convection heat loss coefficient, cover plate to ambient ($Wm^{-3}s$ K^{-1})
V	wind speed (ms^{-1})
h_b	conduction heat loss coefficient through base (Wm^{-2} K^{-1})
k_b	conductivity of insulation (Wm^{-1} K^{-1})
L_b	thickness of base insulation (m)
C_b	conductance of fin to plate bond (Wm^{-1} K^{-1})
$\dot{m}$	total mass flow rate of fluid (kg s^{-1})
C_p	specific heat of water (Jkg^{-1} K^{-1})
Q_{cp}	heat loss from collector absorber to plate (Wm^{-2})
Q_{pa}	heat loss from plate to ambient (Wm^{-2})
Q_{ca}	heat loss from collector to ambient (Wm^{-2})

Theoretical analyses of flat plate collectors have been done in varying degrees of detail for many years and have reached a high degree of maturity (Cadafalch, 2009). These have been able to simulate all aspects of a solar heaters performance. One drawback of the complexity, however, is that it is not possible to see easily how various heater parameters effect the coefficients in relation (2).

In the present work, we show that the standard Hottel–Whillier–Bliss expression for the instantaneous efficiency (Duffie and Beckman, 2006) can be expressed as (2) with each coefficient a straightforward algebraic function of basic solar hot water heater parameters. The advantage of this is that it is possible to check quickly what influence changes of individual heater parameters will have on the efficiency.

As a specific example, we look at the influence of absorber plate absorptance, α, and thermal emittance, ε, on the efficiency. To this end, we show that for reasonably selective absorbers, $\alpha/\varepsilon > 2$, each term in the analytical expressions for η_0, a_1 and a_2 can be split into non-radiative (conduction and convection, subscript C) and radiative (subscript R) parts:

$$\eta_0 = \eta_{0C} - \varepsilon\eta_{0R} \tag{3}$$

$$a_1 = a_{1C} + \varepsilon a_{1R} \tag{4}$$

$$a_2 = a_{2C} + \varepsilon a_{2R} \tag{5}$$

where η_{0C}, η_{0R}, a_{1C}, a_{1R}, a_{2C} and a_{2R} are independent of ε. This is useful because even in the relatively low cost end of the market occupied by flat plate domestic heaters, collectors with selective solar absorbers are almost exclusively used. We can thus extract the α and ε dependences of the efficiency in a simple form which is one of the aims of this work.