Chemical Engineering Science 98 (2013) 317–330

Contents lists available at SciVerse ScienceDirect

Chemical Engineering Science

journal homepage: www.elsevier.com/locate/ces

Analysis of power formulations for the thawing of frozen wood using microwave energy

Fouad Erchiqui*

Department of Applied Sciences, University of Québec at Abitibi-Témiscamingue, Rouyn-Noranda, Canada, J9X5E4

HIGHLIGHTS

- A criterion is proposed to estimate the critical thickness wood above which the Lambert law is valid.
- A model is proposed to analyze the critical thickness of wood as a function of frequency.
- A model is proposed to analyze the critical thickness of wood as a function of temperature and frequency.
- We have studied the thawing of frozen trembling aspen wood using microwave energy.

ARTICLE INFO

Article history:
Received 23 December 2012
Received in revised form
7 March 2013
Accepted 11 March 2013
Available online 11 April 2013

Keywords:
Maxwell's equation
Microwave heating
Sugar maple
Trembling aspen
Yellow birch
White birch

ABSTRACT

This paper addresses the thawing of frozen wood by micro wave. We consider two power formulations for the absorbed energy: the correct power dissipation computed from Maxwell's equations and Lambert's power law equation. The critical thickness above which the two formulations are approximately equivalent is characterized as an exponential-hyperbolic function of frequency and temperature. Four Canadian eastern wood species are used: trembling aspen (*Populus tremuloides*), yellow birch (*Betula alleghaniensis*), white birch (*Betula paperyfera*), and sugar maple (*Acer saccharum*). The nonlinear heat conduction problem involving phase changes such as wood freezing is solved by a volumetric specific enthalpy-based finite element method. Dielectric and thermophysical properties are functions of temperature and moisture content. For illustration purposes, we considered the thawing of frozen trembling aspen wood.

1. Introduction

Microwave irradiation is a technique in which materials with poor thermal and electrical conductivity are heated electrically. Unlike infrared heating, which is strictly a surface phenomenon, dielectric heating rapidly generates heat within the material. In fact, materials with poor electrical conductivity can be heated by microwave only if their molecules are asymmetrically structured. In the case of wood, the application of an electrical field induces asymmetry of the water molecules (polarization). Therefore, for specific frequencies, this causes friction between molecules, which generates heat within the material. Robust experimental and numerical tools are needed to predict the temperature profile. However, only a few published studies have described experimental and mathematical models used to qualify the efficiency of this technique, and many uncertainties of this technique remain to be clarified. For instance, it is unknown whether this technique can penetrate the wood structure (Antti, 1999; James et al., 1985), or whether it can kill microorganisms and insects (Fleming et al., 2005; Lewis et al., 2000; McCullough et al., 2005. Moreover, a number of parameters must be considered to predict microwave penetration into the wood, including the radiation frequency, the wood temperature and moisture content, and the dielectric properties (Fleming et al., 2005).

Microwave heating of frozen wood is a complex process involving highly nonlinear interactions among the mechanical, thermal, and electrical properties. This poses a daunting challenge for numerical simulation, because the combined complexities of heat and mass transfer, phase change, and thermomechanical and electromagnetic interactions must be considered (Brodie, 2007; Rattanadecho and Suwannapum, 2009; Rattanadecho, 2006; Zhu et al., 2007; Datta et al., 1992; Ohlsson and Bengston, 1971; Swami, 1982; Steinhagen and Harry, 1988; Chen et al., 1993; Zhou et al., 1995; Ayappa et al., 1991; Ni and Datta, 2002; Datta and

* Tel.: +1 819 762 0971; fax: +1 819 797 4727.
E-mail address: fouad.erchiqui@uqat.ca

0009-2509/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ces.2013.03.012

Anantheswaran, 2001). Generally, heat generation in wood material is modeled by Lambert's law, which is valid only for semi-infinite samples. A rigorous mathematical formulation of the heating process requires knowing the power flux (Poynting vector) associated with electromagnetic microwave propagation, which is a solution of Maxwell's equation for electromagnetic radiation propagation in dielectric material. In order to model the microwave heating process, three numerical methods are generally used to solve the descriptive equations (finite differences, finite control volume, and finite elements). On this topic, Ohlsson and Bengston (1971) studied one-dimensional heating and solved heat transfer equations in a meat block heated by microwaves. Swami (1982) used a finite difference model to describe heating of gels with high water and NaCl content. Rattanadecho (2006) used a finite control volume discretization method to simulate the heating of liquid layers using a rectangular wave guide and analyzed the influence of frequency and sample size. However, when the medium undergoes a phase change (e.g., solid to liquid), the numerical solution is more difficult due to the presence of one or more moving boundaries of the solid–liquid phases. In general, there are two approaches to solve this type of problem: solve the energy equations for the liquid and solid phases separately, taking into account the moving boundary (solid–liquid interface), or solve the energy equation in terms of the enthalpy function. For microwave heating problems, when a phase change takes place, Panrie et al. (1991) used time-harmonic Maxwell's equations and the enthalpy method to model the microwave melting process for a radially symmetric domain using the finite difference method. Bhattacharya et al. (2002) analyzed thawing of 1D slabs and 2D cylinders in the presence of volumetric heat sources due to microwave propagation within the samples by using the finite element based enthalpy formulation. Basak and Ayappa (1997) used the effective heat capacity method to analyze microwave thawing of materials. Coleman (1990) studied microwave melting in one space dimension.

In the present study, we examine the effect of constant attenuation, frequency, and temperature on critical wood thickness for the applicability of Lambert's law in finite sapwood of four Canadian eastern wood species: trembling aspen (*Populus tremuloides* Michx), yellow birch (*Betula alleghaniensis*), and sugar maple (*Acer saccharum*). We consider two power formulations: the correct power dissipation computed from Maxwell's equations and Lambert's power law equation. The critical thickness above which the two formulations are approximately equivalent is then characterized as an exponential-hyperbolic function of frequency and temperature. The nonlinear heat conduction problem involving phase changes such as wood freezing is solved by a volumetric specific enthalpy-based finite element method. Dielectric and thermophysical properties are functions of temperature and moisture content (Kanter, 1957). For illustration purposes, we examine the thawing of frozen trembling aspen wood.

2. Microwave energy absorption and Poynting's theorem

The time-dependent power flow density of an electromagnetic wave is given by the instantaneous Poynting vector **S** (Pozar, 2011):

$$\boldsymbol{S} = \frac{1}{2}\boldsymbol{E} \times \boldsymbol{H}^* \tag{1}$$

where **E** and $\boldsymbol{H}^*$ are the electric field (V m^{-1}) and the conjugate magnetic field (A m^{-1}), respectively. For an isotropic dielectric material, we have (see Appendix I):

$$\nabla \cdot \boldsymbol{S} = -j\frac{\omega}{2}(\mu\, \boldsymbol{H}\cdot\boldsymbol{H}^* + \varepsilon^*\boldsymbol{E}\cdot\boldsymbol{E}^*) \tag{2}$$

where ω (rad/s^{-1}) is the frequency of the incident radiation, ε^* (F m^{-1}) is the complex conjugate of the dielectric permittivity ε and μ (Henries m^{-1}) is the complex permeability:

$$\varepsilon(\omega) = \varepsilon'(\omega) - j\varepsilon''(\omega) \quad \text{and} \quad \mu(\omega) = \mu'(\omega) - j\mu''(\omega) \tag{3}$$

The real part of the $\varepsilon(\omega)$ and $\mu(\omega)$ represents the material's ability to stored electrical and magnetic energy, respectively, whereas their imaginary part represents the electrical energy dissipation. The negative sign on the right-hand side of (3) indicates that the amount of energy in the volume decreases if the net amount of power flowing through the surface is positive. The power dissipated per unit volume is given by (see Appendix I):

$$P^{\max} = -Re(\nabla \cdot \boldsymbol{S}) = \frac{1}{2}\omega\varepsilon''|\boldsymbol{E}|^2 \tag{4}$$

In wood materials, the magnetic permeability $\mu(\omega)$ is generally approximated by its value μ_0 in free space. With this assumption and the electroneutrality of wood $(\nabla\cdot(\nabla\cdot E) = 0)$ we deduce, from the Maxwell's equations the expression of Helmholtz's equation of wave propagation:

$$\nabla^2\overline{\boldsymbol{E}} - \gamma^2\overline{\boldsymbol{E}} = 0 \tag{5}$$

where $\overline{\boldsymbol{E}}(\boldsymbol{r}) = e^{+i\omega t}[\boldsymbol{E}(\boldsymbol{r}, \boldsymbol{t})]$ and γ is the constant complex propagation:

$$\gamma = \alpha + j\beta \tag{6}$$

β is the attenuation constant and α is the phase (a constant). These parameters are related to the dielectric properties of the material and frequency of radiation by:

$$\alpha = \frac{\omega}{c}\sqrt{\frac{\varepsilon'}{2}\left(\sqrt{1+\tan^2\delta}+1\right)} \tag{7}$$

$$\beta = \frac{\omega}{c}\sqrt{\frac{\varepsilon'}{2}\left(\sqrt{1+\tan^2\delta}-1\right)} \tag{8}$$

$c = 1\sqrt{\mu_0\varepsilon_0}$ is the speed of light (ε_0 is the dielectric permittivity ($=8.8541 \times 10^{-12}$ F/m) of the free space). The term δ ($= tg^{-1}(\varepsilon''/\varepsilon')$) is the dielectric loss angle. The attenuation (a constant), β, controls the rate at which the incident field intensity decays into a simple. β^{-1} is known as the penetration depth. The phase (a constant) α represents the change of phase of the propagation radiation and is related to the wavelength of radiation by $\lambda = 2\pi/\alpha$.

2.1. Uniform plane wave propagation and power dissipation

Consider one-dimensional propagation energy through the thickness 'L' of the sample wood material. The incident microwave is assumed to be normal on opposite face of the sample. (see Fig. 1). The wave has only a z-component of electric field which is a function of the parameter y. Eq. (5) is reduced to the following:

$$\frac{d^2E_z}{dy^2}\boldsymbol{a}_z - \gamma^2E_z\boldsymbol{a}_z = 0, \quad for \;\; 0 \le y \le L \tag{9}$$

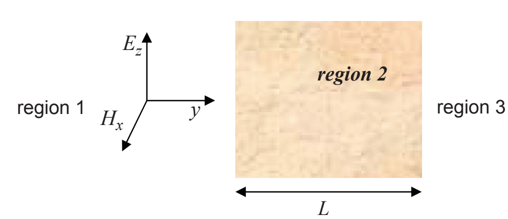

Fig. 1. Schematic of layered sample exposed to plane microwave.