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Electronic band gaps and transport in Cantor graphene superlattices

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ABSTRACT

The electronic band gap and transport in Cantor graphene superlattices are investigated theoretically. It is found that such fractal structure can possess an unusual Dirac point located at the energy corresponding to the zero-averaged wave number (zero—$\overline{k}$). The location of the Dirac point shifts to lower energy with the increase of order number. The zero—$\overline{k}$ gap is robust against the lattice constants and less sensitive to the incidence angle. Moreover, multi-Dirac-points may appear by adjusting the lattice constants and the order, and an expression for their location is derived. The control of electron transport in such fractal structure may lead to potential applications in graphene-based electronic devices.

1. Introduction

The electronic band gap and transport in graphene have attracted much attention due to its novel electronic properties since the extremely thin carbon material was experimentally realized by Novoselov et al. in 2004 [1]. In graphene, the quasiparticles close to the Dirac point (DP) (often referred to as K and K') of the Brillouin zone posses a gapless linear energy spectrum. As a consequence, graphene exhibits numerous unique electronic and transport properties, such as half-integer quantum Hall effect [2–4], the minimal conductance [2], the zitterbewegung [5,6], Klein tunneling [7] and atomic collapse [8,9].

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Since superlattices are very successful in controlling the electronic transport, more and more theoretical [10–30] and experimental [31–33] researches focus on the graphene superlattices (GSLs) with electrostatic potential [10–25] and magnetic barriers [26–30]. Recently, several researches about the zero-averaged wave number (zero—$\overline{k}$) gap associated with the new DP in GSLs have been reported [11–23,26]. The zero—$\overline{k}$ gap in GSLs is similar to the zero-averaged refractive index band gap in photonic crystals containing left-handed metamaterials [34]. Contrary to Bragg gaps, the zero—$\overline{k}$ gap associated with the new DP is robust against the lattice constant, structural disorder [11], and external magnetic field [26], which can be applicable to control the electronic transport in GSLs. The electronic band gaps and transport in quasi-periodic GSLs, such as Fibonacci [17], Thue–Morse [18–20], and double-periodic [21] GSLs, have also been investigated theoretically. It is found that the zero—$\overline{k}$ gap also exists in these quasi-periodic GSLs, it is less sensitive to the incidence angle, and robust against the lattice constants. The electronic band gaps and transport properties in these quasi-periodic GSLs may facilitate the development of graphene-based electronic devices.

Cantor set, a fractal structure [35], is a homogeneous and self-similar geometric object, which can be used to describe many physical phenomena. As the simplest fractal structure, the triadic Cantor set has been widely studied on the transmission properties of light and electrons [36–38], quantum scattering and tunneling [39,40]. The Cantor set is different from the periodic and quasi-periodic sequences, such as Fibonacci, Thue–Morse, and double-periodic sequence. It is also interesting to study the electronic band gaps and transport in Cantor GSLs. Sun et al. [41] have studied the transmission of electrons tunneling through GSLs with fractal and periodic magnetic barriers. Most recently, Liu et al. [42] have investigated the spin-polarized transport properties of electrons tunneling through a fractal graphene superlattice sandwiched between two ferromagnetic graphene electrodes, and the spin conductance and the tunneling magnetoresistance of the ferromagnetic graphene/fractal graphene superlattice/ferromagnetic graphene junction have been compared with those of the ferromagnetic graphene/symmetric periodic graphene superlattice/ferromagnetic graphene junction. Motivated by these unusual electronic properties in the periodic and quasi-periodic GSLs, we investigate the electronic band gaps and transport in the graphene-based Cantor sequence. In this paper, we find that the zero—$\overline{k}$ gap and DP also exist in the graphene-based Cantor sequence. Different from the Fibonacci case, the Cantor sequence do not visualize the Cantor-like distribution of energy bands. And the location of DP shifts to lower energy region as the order n increases. We also derived an expression for the location of multi-Dirac-points.

2. Theoretical model

A triadic Cantor sequence can be out-bound with the single layer width is given, and the n-th triadic Cantor sequence is defined by the rule: $S_n = S_{n-1}B_nS_{n-1}$ for $n \geqslant 2$, while $S_0 = \mathrm{A}$, $S_1 = \mathrm{ABA}$, as shown in Fig. 1(a). B_n for n-th layer denotes the layer B with a width of $d_{\mathrm{B}_n} = 3^{n-1}d_{\mathrm{B}}$. Here, the element A (B) denotes a barrier V_{A} (well V_{B}) with the width $d_{\mathrm{A}}(d_{\mathrm{B}})$. The triadic Cantor sequence also can be in-bound with the total width of the structure is given at the beginning, as shown in Fig. 1(b). First take a seed to be a bulk of layer A with a given width (in Fig. 1(b), the initial width is the width of 2-th Cantor structure in out-bound form). Then divide the seed into three equal parts, and replace the center part of the seed with layer B. Then repeat the same procedure over all remaining initial layer, as if they were seeds. And an n-th in-bound form Cantor structure is obtained when these steps recurred n times. It is notable that the ratio of width of layer A and B, $d_{\mathrm{A}}/d_{\mathrm{B}}$, is always equal to 1 in in-bound form, while it can be adjusted in out-bound form. $N_{\mathrm{A}} = 2^n$ and $N_{\mathrm{B}} = 3^n - 2^n$ are the numbers of barrier A and well B for the n-th Cantor sequence both in in-bound and out-bound form. In this work, we will focus on the out-bound form.

The Dirac-like electrons moving inside a monolayer graphene in the presence of the electrostatic potential $V(x)$ can be described by the Dirac-like equation:

$$[-i\hbar v_F\vec{\sigma}\cdot\nabla + V(x)]\Psi(x,y) = E\Psi(x,y), \tag{1}$$

where $\vec{\sigma} = (\sigma_x, \sigma_y)$ are the Pauli matrices, $v_F = 10^6$ m/s is the Fermi velocity, and $\Psi = (\tilde{\psi}_{\mathrm{A}}, \tilde{\psi}_{\mathrm{B}})^T$ is a two-component pseudospinor wave function. Due to the translation invariance in the y direction, the wave