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A two-pressure model for slightly compressible single phase flow in bi-structured porous media

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HIGHLIGHTS

- Upscaling of slightly compressible single phase flow in bi-structured porous media.
- The resulting macroscopic system is a two-pressure equations.
- All the effective coefficients are entirely determined by three closure problems.
- Comparison with pore-scale direct numerical simulations for a particle filter.

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ABSTRACT

Problems involving flow in porous media are ubiquitous in many natural and engineered systems. Mathematical modeling of such systems often relies on homogenization of pore-scale equations and macroscale continuum descriptions. For single phase flow, Stokes equations at the pore-scale are generally approximated by Darcy's law at a larger scale. In this work, we develop an alternative model to Darcy's law that can be used to describe slightly compressible single phase flow within bi-structured porous media. We use the method of volume averaging to upscale mass and momentum balance equations with the fluid phase split into two fictitious domains. The resulting macroscale model combines two coupled equations for average pressures with *regional* Darcy's laws for velocities. Contrary to classical dual-media models, the averaging process is applied directly to Stokes problem and not to Darcy's laws. In these equations, effective parameters are expressed via integrals of mapping variables that solve boundary value problems over a representative unit cell. Finally, we illustrate the behavior of these equations for model porous media and validate our approach by comparing solutions of the homogenized equations with computations of the exact microscale problem.

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1. Introduction

Porous media are intrinsically highly complex materials, with the consequence that transport phenomena generally occur over a broad spectrum of spatial and temporal scales. Even for single phase flow, this variety of characteristic time and length scales may preclude the use of a one-equation continuum representation. For instance, advection and diffusion of a single species in a system with stagnant zones or dead-end pores are better represented macroscopically by a two-equation model in which the species concentration is divided into mobile and immobile fractions (see Coats and Smith, 1964 for an early discussion on the subject). In many applications (including flow in fractured media, automobile soot filters or chemical and biochemical reactors), the porous medium itself exhibits a distinct two-region topology, e.g., as a consequence of a contrast of porosity or a difference in the pore structure geometry. Herein, we will use the term bi-structured to describe these porous media, a term which represents a more general definition than the traditional dual-media or dual-porosity terminology. With this definition, one may differentiate each region according to a number of different properties including the topology of the fluid flow. For example, in fractured media, fractures represent a zone of preferential flow whereas the amplitude of the velocity field in the matrix blocks is often orders of magnitude smaller. In the literature, solute transport in such systems is often described using mobile/immobile models. Rapid advective transport in the mobile domain is accompanied by diffusive mass transfer of the solute in the immobile domains. This contrast of time scales may strongly impact the concentration field and it is well known that breakthrough curves, in such configurations, typically exhibit strong tailing effects.

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More generally, if time and length scales characterizing the two regions differ significantly, non-equilibrium models may be mandatory. An example of one such model is a generic two-equation formulation (see Coats and Smith, 1964; Brusseau and Rao, 1990) in which average concentrations are defined over each region separately. In this model, each equation involves the average velocity within each region; velocity fields that are also known as "regional velocities". The situation simplifies for mobile/immobile systems since the regional velocity of one region is negligible and, therefore, the net superficial velocity corresponds to the superficial velocity of the mobile region. However, bi-structured systems are not necessarily of the mobile/immobile type. If advection cannot be neglected in the slower region, a mobile/mobile model (Skopp et al., 1981; Gerke and VanGenuchten, 1993; Ahmadi et al., 1998; Cherblanc et al., 2003) with two different regional velocities may be necessary. In practice, experimental measurements of these regional velocities are difficult and one can often access only the total imposed filtration velocity. Regional velocities may therefore be determined indirectly by inverse optimization techniques, although such approaches will be primarily useful in large-scale 1D cases. For interpreting a complete 3D macroscale problem, the momentum transport equations are needed along with mass transport equations. This issue has been addressed theoretically in Quintard and Whitaker (1996) using the volume averaging technique. In this cited paper, large-scale momentum transport equations are determined via a two-step upscaling procedure: Stokes equations are first averaged to obtain a Darcy-scale description within each region and, then, a regional averaging is performed in order to obtain the large-scale equations. This was done in Quintard and Whitaker (1996) for the flow of a slightly compressible fluid and led to a large-scale two-equation model involving two average pressures; a result thus generalizing the classical two-equation model of Barenblatt et al. (1960). Further, average velocities can be determined via regional Darcy's laws in which regional permeability tensors are expressed as integrals of mapping variables that solve the so-called closure problems defined at the Darcy-scale (see Quintard and Whitaker, 1998 and Fig. 1). Again, this derivation is a recursive procedure based on a successive averaging from the pore-scale to the Darcy-scale and then to the large-scale. Typically, the following constraints must be satisfied:

1. The pore-scale characteristic length must be much smaller than the characteristic lengths of the two regions (separation of scales), so that Stokes can be upscaled to Darcy's law within each region.
2. The subsequent upscaling from Darcy's law within each region to a large-scale Darcy's law or a dual-media model (as developed in Quintard and Whitaker, 1996) also requires a separation of scales between the regional and large-scale characteristic lengths.

Therefore, this two-region approach applies only to large systems and cannot be used directly for some bi-structured porous media at the pore-scale, for which the first separation of scales does not hold. In this work, our goal is to derive one such two-pressure model *directly* from the Stokes problem at the pore-scale.

There are many industrial applications involving bi-structured porous media where it may be useful to split the flow of a single phase into two coupled continuum equations. This is the case, for instance, in tangential filters in which two sets of channels are exchanging via small holes or porous walls (Belfort et al., 1994; Zeman and Zydney, 1996; Oxarango et al., 2004; Borsi and Lorain, 2012). Recently, in an attempt to model the liquid distribution within structured packings used in chemical engineering processes, Mahr and Mewes (2008) have found convenient to split the (physically homogeneous) liquid phase into two fictitious phases. This approach was motivated by the fact that the structured packings are made of an assembly of corrugated sheets where two-adjacent sheets are inclined by a given angle with respect to the vertical axis and the opposite of this angle, respectively. As a consequence, the liquid phase behaves as if split into two pseudo-phases flowing along each sheet with a preferential direction. These phases are not (except perhaps at very low saturation) completely independent since adjacent sheets are in contact and the wetting liquid can flow from one sheet to the other. In the paper referenced above, this transfer between the two liquid phases is treated using a heuristic function involving the difference between the volume fraction of fluid in each phase. Although

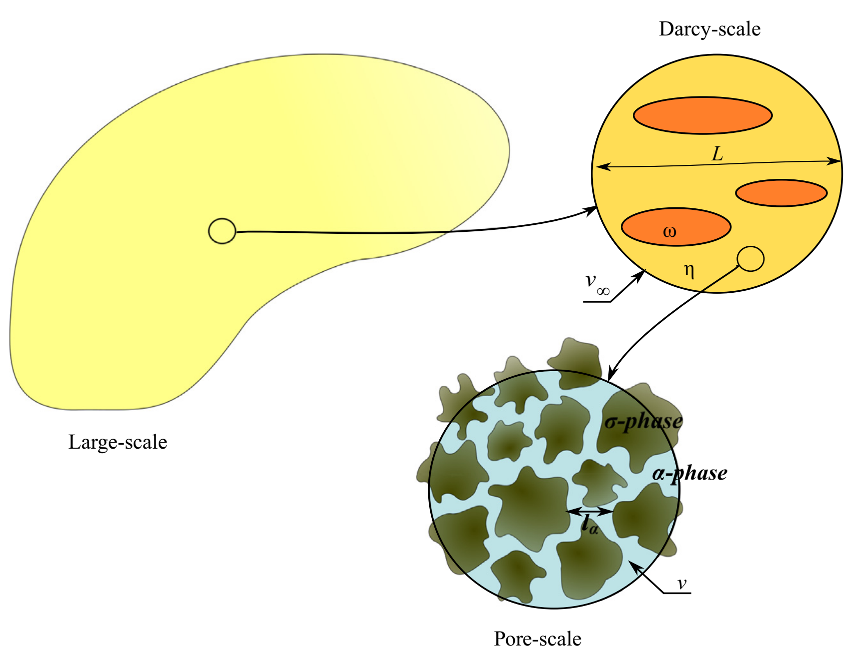

Fig. 1. Schematic representation of the hierarchy of length scales of a classical dual-porous medium as presented in Quintard and Whitaker (1996).

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