Superlattices and Microstructures 73 (2014) 82–97

Contents lists available at ScienceDirect

Superlattices and Microstructures

journal homepage: www.elsevier.com/locate/superlattices

Spin-wave theory applied to the low-temperature properties of the spin-1/2 ferromagnetic chain with the *y*-direction exchange anisotropy

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ARTICLE INFO

Article history:
Received 18 January 2014
Received in revised form 17 April 2014
Accepted 9 May 2014
Available online 27 May 2014

Keywords:
Ferromagnetic chain
Spin-wave theory
Exchange anisotropy

ABSTRACT

In this paper, we apply the spin-wave theory within the simple self-consistent method, and provide a simple way to investigate the spin-1/2 ferromagnetic chain with the *y*-direction exchange anisotropy. It is found that the anisotropy has an influence on the ground-state and low-temperature properties of the system. Anisotropy dependences of both the susceptibility maximum and specific heat maximum fit well to the exponential laws. The linear decrease is shown for the position of the susceptibility maximum. The position of the specific heat maximum displays three kinds of the anisotropy dependence, i.e., the power law, constant law and exponential law. The specific heat coefficient is calculated to be a monotonically increasing function of the anisotropy in the zero-temperature limit. At some low temperature, the Heisenberg behavior is observed to cross over the XY behavior, which agrees with the experimental result of the quasi-one-dimensional compound $(C_6H_{11}NH_3)CuBr_3$. In the isotropic and anisotropic cases, our results are in good accord with the exact Bethe-ansatz results and quantum Monte Carlo estimates, and have some advantages over the findings of Green-function method and the renormalization group method at low temperatures.

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http://dx.doi.org/10.1016/j.spmi.2014.05.016
0749-6036/

1. Introduction

Recently, the theoretical studies of one-dimensional quantum Heisenberg ferromagnetic models are spurred by the experimental progress in the synthesis of ferromagnetic compounds [1]. Some quasi-one-dimensional ferromagnets which can be described by the Heisenberg model, are the $CuCl_2$ sulfoxide complexes [2], organic p-NPNN compounds [3] and cuprates TMCuC [4,5]. Many methods have been devoted to interpret the thermodynamic properties of one-dimensional ferromagnetic systems. Spin wave theory [6], Schwinger-boson mean-field theory [7], Monte Carlo method [8], renormalization group method [9], Cluster series expansions [10], and the Green's function method [11], are some of the methods used in these studies.

The spin-wave theory which was developed by Anderson [12] and Kubo [13], is a powerful theoretical tool for the low-temperature properties of Heisenberg magnets. However, the conventional spin-wave theory encounters with the thermodynamic divergences in the low-dimensional cases. In order to overcome these divergences, Takahashi [14] formulated a modified spin-wave theory for low-dimensional isotropic Heisenberg ferromagnets by constraining the total magnetization to be zero. Recently, his idea has been applied to more complex systems, such as frustrated magnets [7,15,16], random magnets [17], entanglement magnets [18], and spatially anisotropic magnets [19].

In this paper, we will study the low-temperature properties of the spin-1/2 ferromagnetic chain by using the spin-wave theory within Takahashi's idea. The system considered is characteristic of the intrachain interaction containing small y-direction anisotropy. It is modeled as the one-dimensional spin-$S = 1/2$ quantum ferromagnetic Heisenberg model. One of the best realization of this system is the bromine compound $(C_6H_{11}NH_3)CuBr_3$ [20], where the intrachain interaction exceeds the interchain interaction by three orders of magnitude. It was argued [20] that the small xy anisotropy of the intrachain interaction is due to the symmetry of the local environment for the Cu^{2+} ions.

It is noted that the existing schemes of implementing Takahashi's idea are minimizing the free energy by introducing an ideal spin-wave density matrix. However, these schemes are involved into the complicated calculations. In this paper, our aim is to provide a simple self-consistent method for the ferromagnetic Heisenberg chain, which introduces a Lagrange multiplier in the Hamiltonian to keep zero magnetization. Our results show that the anisotropy has an influence on the ground-state and low-temperature properties of the system. In both isotropic and anisotropic cases, our results agree with the exact Bethe-ansatz results [21–24]. Our low-temperature results of the isotropic system are content with the quantum Monte Carlo (QMC) estimates [11]. It is found that our method can give the reliable results about thermodynamic properties of the ferromagnetic chain.

The rest of the paper is organized as follows: In Section 2, we rewrite the Hamiltonian. We introducing a Lagrange multiplier, and diagonalize the effective Hamiltonian in terms of bosonic operators by means of the Holstein–Primakoff transformation [25] combining with Bogoliubov transformation [26]. In Section 3, we present the results of the numerical calculations of the ground-state and finite-temperature properties for the ferromagnetic chain with the anisotropy. Finally, the main points of the work are summarized in Section 4.

2. Model

In this section, we will employ spin-wave method to study the spin-$S = 1/2$ quantum ferromagnetic chain with the y-direction exchange anisotropy. A well-known example of this chain is the quasi-one-dimensional compound $(C_6H_{11}NH_3)CuBr_3$ [20], which is built up from the ferromagnetic Cu^{2+} ($S = 1/2$) chains. The intrachain interactions are predominant in the compound, with small easy-plane exchange anisotropy. The system can be described by the one-dimensional spin-$S = 1/2$ quantum ferromagnetic Heisenberg model with the y-direction exchange anisotropy. Its Hamiltonian is

$$H = -J\sum_{\langle i,j\rangle}\left(S_i^x S_j^x + pS_i^y S_j^y + S_i^z S_j^z\right). \tag{1}$$

Here J is the nearest-neighbor ferromagnetic exchange interaction with $J > 0$, and p denotes the y-direction exchange anisotropy with $0 \leqslant p \leqslant 1$. For $p = 1$ and 0, the system stands for the isotropic