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Excitations and superfluidity in non-equilibrium Bose–Einstein condensates of exciton–polaritons

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Abstract

We present a generic model for the description of non-equilibrium Bose–Einstein condensates, suited for the modelling of non-resonantly pumped polariton condensates in a semiconductor microcavity. The excitation spectrum and scattering of the non-equilibrium condensate with a defect are discussed.

Keywords: Bose–Einstein condensation; Superfluidity; Polaritons

1. Model and elementary excitation spectrum

Due to the finite polariton lifetime, the nature of polariton condensates [1] is fundamentally different from that of their superfluid ^{4}He and atomic Bose–Einstein condensate counterparts: the polariton condensate has to be constantly replenished and arises as a dynamical equilibrium between pumping and decay. The theoretical modelling of such systems in which interactions, coherence, pumping and decay are equally important poses a challenge that was taken up only recently [2–4]. We will present here our mean field model of such condensates [4] and use it to study the excitation spectrum of a homogeneous non-equilibrium condensate and the problem of a small defect moving through it.

Our model does not include any details of the specific relaxation mechanisms of high-energy polaritons into the condensate, but is only based on some general assumptions: (i) a single state of lower polaritons is macroscopically occupied so that it can be described by a classical field; (ii) The momentum space can be divided into two parts: one part at small momenta where the coherence is important and one part at large momenta, where it is negligible. The polaritons in the

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high-momentum states act as a reservoir that replenishes the condensate. Under typical excitation conditions, the wave vector scale to separate both systems should be chosen to be of the order of a few μm^{-1}; (iii) The state of the reservoir is fully determined by its spatial polariton density $n_R(x)$. This last assumption requires that the reservoir polariton momentum distribution reaches some stationary state in momentum space. Under these assumptions, the condensate dynamics is to a first approximation described by a generalized Gross–Pitaevskii equation including loss and amplification terms

$$\mathrm{i}\frac{\partial\psi}{\partial t} = \left\{-\frac{\hbar\nabla^2}{2m} + \frac{\mathrm{i}}{2}[R(n_R) - \gamma] + g|\psi|^2 + 2\tilde{g}n_R\right\}\psi, \tag{1}$$

where m is the lower polariton mass, γ its decay rate and g is the strength of the polariton–polariton interaction within the condensate. The stimulated scattering of reservoir polaritons into the condensate is modelled by the term $R(n_R)$ and the mean field interaction experienced by the condensate polaritons due to elastic collisions with the reservoir polaritons is given by $2\tilde{g}n_R$. The description (1) of the polariton condensate in terms of a deterministic classical field requires its density and phase fluctuations to be small. This regime is reached for pump powers well above the condensation threshold. The equation for the condensate dynamics is coupled to a diffusion equation for the reservoir polaritons

$$\frac{\partial n_R}{\partial t} = P - \gamma_R n_R - R(n_R)|\psi(x)|^2 + D\nabla^2 n_R, \tag{2}$$

where P is the pump rate due to external laser, γ_R is the reservoir damping rate and D its diffusion constant.

The stationary state and the elementary excitation spectrum of Eqs. (1) and (2) are discussed in Ref. [4]. In case the reservoir damping rate γ_R is much larger than the condensate polariton damping rate γ, the condensate excitation spectrum is of the form

$$\omega_\pm(k) = -\frac{\mathrm{i}\Gamma}{2} \pm \sqrt{\omega_{\mathrm{Bog}}(k)^2 - \frac{\Gamma^2}{4}}. \tag{3}$$

Here, $\omega_{\mathrm{Bog}} = [(k^2/2m+2\mu)(k^2/2m)]^{1/2}$ is the usual Bogoliubov dispersion of dilute Bose gases at equilibrium. The non-equilibrium nature of the system is quantified by the effective relaxation rate $\Gamma = \zeta\gamma$, where ζ depends on the pumping rate and on the functional form of $R(n_R)$ [4]. The most important differences with the excitation spectrum of equilibrium condensates is the non-vanishing imaginary part of $\omega(k)$ for all $k \neq 0$ and the flatness of its real part for small k. The '+'-branch is diffusive for small wave vectors. A similar excitation spectrum was found in Ref. [2] for a specific model of non-equilibrium condensation, within a completely different approach, indicating that the form (3) is a general result for non-equilibrium condensates.

Our model is also straightforwardly applied to the Josephson oscillations between two condensates connected by quantum mechanical tunneling. The frequency of the density oscillations between the two wells is given by Eq. (3) with ω_{Bog} replaced by the equilibrium Josephson frequency [8].

2. Flow past a defect

One of the benchmark properties of condensed Bose systems is superfluidity. As a first step in the study of superfluidity in non-equilibrium systems, we will discuss the scattering of a moving