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A study of mixing by chaotic advection in two three-dimensional open flows

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HIGHLIGHTS

- Mixing by chaotic advection is studied in two types of 3-D open flows.
- In one geometry—the Kenics KM static mixer—the flow is spatially periodic.
- In the other geometry—the 3-D confocal elliptic mixer—the flow is time periodic.
- Symmetry can provoke or inhibit chaotic advection depending on the flow.

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ABSTRACT

Mixing by chaotic advection is studied in two types of three-dimensional open flows. The first flow is spatially periodic, it consists of a cylindrical tube with a finite number of static mixer elements inside. Mixing in this 3-D flow with no moving parts is studied by numerically solving the momentum equations using computational fluid dynamics. Using Lagrangian particle tracking simulations, we determine the number of elements necessary to mix a blob of passive tracer over the entire exit cross-section of the mixer and to examine the influence of the initial blob location. The second mixer considered here is the time-periodic 3-D flow between two confocal elliptic cylinders whose inner and outer boundaries glide at constant or variable velocity. This flow consists of axial Poiseuille flow superimposed on the 2-D cross-sectional flow between two concentric, confocal ellipses whose walls can glide along their elliptical circumference while keeping the geometry invariant. This flow has moving parts, but induces a lower pressure drop and, as we shall demonstrate, can be controlled by varying the motion of the two boundaries or by modifying the axial flow rate. It is shown that this boundary motion is feasible by comparing streaklines obtained from experiments to numerical results based on the analytical solution of the Stokes equations for this flow. It is also shown that there is a certain range of modulation frequencies for which mixing is enhanced; outside of this range the flow is practically regular.

1. Introduction

Chaotic advection or Lagrangian turbulence is characterized by chaotic trajectories of a material element of fluid, as in turbulent flow, in a flow dominated by viscous forces. It is now well known that chaotic advection can only occur in certain two-dimensional (2-D) time-varying flows and in both spatially (steady) and time-periodic, three-dimensional (3-D) flows.

In Aref's (1984) pioneering work the mixing created by two blinking vortices in a layer of inviscid fluid was studied analytically. Since then numerous experimental, theoretical, and numerical studies on 2-D periodic flows followed. A simple flow that can exhibit chaotic advection is the one between two eccentric, rotating cylinders. When the cylinders turn at constant velocities and in opposite directions, a saddle point is formed in the region of minimum gap. Chaotic advection occurs when the saddle point is perturbed, for example by turning one cylinder at a periodic angular velocity.

The theoretical analysis of this 2-D flow—usually called the journal bearing flow—is simplified by the fact that an analytical solution exists for the cases of constant angular velocities of both cylinders. The analytical solution is available in bipolar coordinates (Jeffery, 1922; Ballal and Rivlin, 1976), in modified bipolar coordinates (Di Prima and Stuart, 1972), and in a mixed Cartesian–cylindrical coordinate system (Wannier, 1950).

Numerous tools were developed to study chaotic advection in the journal bearing flow: Poincaré sections (Aref and Balachandar, 1986), the location of low-order periodic points, stretching calculations

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(Swanson and Ottino, 1990), and the potential mixing zone (Kaper and Wiggins, 1993), amongst others.

A similar 2-D flow, the one between two confocal gliding ellipses, was imagined, constructed, and analyzed by our group (Saatdjian et al., 1994, 1996). In this flow the boundaries glide along their elliptical circumference, the elliptical trajectories are similar to those of planets around the sun. The journal bearing flow discussed above and the flow between confocal gliding ellipses are two different deformations of circular Couette flow. Surprisingly, both models have been used in ancient times to model the trajectories of planets around the sun after the circular orbits around the sun in the center model of Aristarchus were shown to be inconsistent with measurements.

The analytical solution when both elliptical boundaries glide at constant velocity, but in opposite directions, shows the appearance of two saddle points joined by two different streamlines in the regions of minimum gap; this is a heteroclinic trajectory. Since chaotic mixing is created by perturbing saddle points, this geometry with two saddle points was considered to be particularly effective.

Mixing in this 2-D flow was studied using several tools, such as Poincaré sections, by tracking the transport of a passive scalar (Saatdjian et al., 1996), and, in particular, by calculating the heat transfer enhancement, as a function of geometry and modulation frequency, for the time-averaged total heat flux across the elliptical gap (Mota et al., 2007). The significance of this tool is explained later.

Chaotic advection in a steady, spatially periodic 3-D flow was imagined by Jones et al. (1989); it consists of the flow in a twisted pipe. A heat exchanger based on this design was constructed by Le Guer and Peerhossaini (1991) and led to moderately higher heat-transfer coefficients without a pressure drop increase.

The wide use of static mixers in the chemical industry has led to numerous studies on the flow through these devices (Hobbs et al., 1997; Hobbs and Muzzio, 1997, 1998a–c; Szalai and Muzzio, 2003). New devices that can mix highly viscous fluids by chaotic advection in spatially periodic flows have recently been tested (Metcalfe and Lester, 2009). Since the flow field in most static mixers is rather complex, the available tools for the study of chaotic advection all require a CFD code to solve the flow equations, which make the analysis computationally more expensive than for simpler prototype flows.

Experiments on 3-D chaotic flows were performed by Kusch and Ottino (1992); these authors were concerned with a partitioned pipe mixer (spatially periodic) and with the eccentric helical annular mixer (time periodic), which is simply the 2-D journal bearing flow (i.e, the flow between two eccentric rotating cylinders) with a superimposed unidirectional axial Poiseuille flow. The theoretical analysis for this 3-D version of the journal bearing flow is more complicated because of the additional parameters involved, mainly the axial velocity. There are no fixed points in this flow and most of the tools used in Hamiltonian systems (2-D periodic flows) are either not applicable or of little use. A recent analysis (Rodrigo et al., 2003a, 2006; Mota et al., 2008) for time-periodic 3-D flows showed that there is a certain range of modulation frequencies for which the streaklines become chaotic; above or below this interval the flow is quite regular.

The numerous experimental and computational examples discussed above have shown that in continuous throughput flows (i.e., fluid continuously flowing in a tubular device), chaotic particle trajectories can be generated by superimposing a time-variable 2-D cross-sectional flow, generated by boundary rotation, over the axial flow, or by rendering the flow spatially periodic through insertion of internal mixing elements. Furthermore, it is necessary (but not sufficient) that the superimposition of the cross-sectional streamline portraits, taken at arbitrary times in the former case, or at arbitrary axial locations in the latter case, show streamline crossing (Ottino et al., 1992).

In this paper we study two different 3-D chaotic flows: the spatially periodic flow in a static mixer and the time-periodic flow between two confocal gliding elliptic cylinders. Thus, Lagrangian turbulence is achieved by spatial changes in one case and by time modulation in the other. The analysis of the spatially periodic flow is based on the numerical solution of the momentum equations using an open source CFD code. Lagrangian methods are used to study the influence of the number of elements and of the location of the injection point.

The second 3-D open flow studied here is the one between two confocal elliptic cylinders whose boundaries glide along their circumference so that the cross-section is unchanged. For this flow we have at our disposal an analytical solution when the boundaries glide at constant velocity. Assuming that the Strouhal number is small, we solve numerically the transport equation for a passive scalar to determine the boundary displacement protocol that will yield the best mixing for a given geometry. The limiting case, when the axial flow is zero, can in some instances guide the user in the choice of some geometrical parameters. We also present a method that allows the optimization of the geometry of this mixer and the modulation protocol.

2. Mixing in a spatially periodic 3-D flow

There are many different types and geometries of static mixers that are manufactured worldwide. These static mixers are inserted inside a tube in order to enhance mixing, heat transfer, and eventually chemical reaction in the flow. The geometry considered here is based on the Kenics KM static mixer manufactured by Chemineer (Chemineer, 1997). The Kenics mixer has helicoidal twisted tape inserts; these are illustrated in the schematic of Fig. 1. The tapes are twisted through 180° and abut one another at a 90° angle, providing successive right- and left-hand rotation of the fluid elements.

The main characteristics of the mixer considered in the present study are given in Table 1. The length-to-diameter ratio (L/D) of each twisted tape element is 1.5, which is the value for most commercial Kenics mixers. The plate thickness is 0.3175 cm, yielding a thickness-to-diameter ratio of 1/16. The mixer consists of six elements tightly inserted into a pipe with an entrance and exit zones of empty tube; the length of each of these zones is equivalent to two pipe diameters. Thus, the overall length of the pipe is $2\times(2D)+6\times(1.5D)=13D$.

The momentum equations were solved numerically using OpenFOAM®, an open source CFD toolbox (OpenCFD Ltd., 2010). This code uses a control-volume formulation to solve the primitive

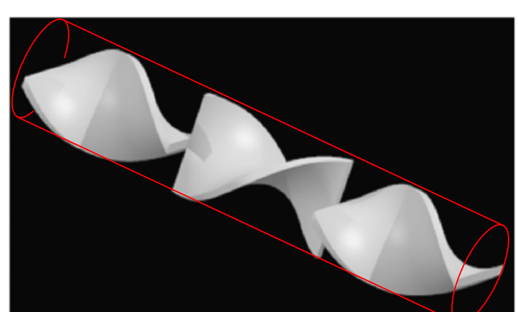

Fig. 1. Standard helical twisted tape inserts of the Kenics static mixer. Here, $L/D=1.5$, the twist angle per element is 180°, and the elements abut at 90°.