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Rarefaction wave propagation in tapered granular columns

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ABSTRACT

The study of stress waves in granular materials is of some importance in several industrial bulk materials handling systems. In particular, the growth in the magnitude of rarefaction waves due to friction has been thought to play an important role in the silo quaking phenomenon. To this end, this paper examines the propagation of rarefaction waves in granular columns subject to Coulomb wall friction, focusing on the effect of geometry by examining converging and diverging tapered columns. A one-dimensional dynamic model of these systems is developed and analytical solutions of this model are compared to a numerical model based on the Arbitrary Lagrangian–Eulerian formulation in a finite element analysis. This numerical model was first validated using the known behaviour of cylindrical columns. In all cases, the rarefaction waves examined in this work grew with the distance travelled up the column; however, the rate was shown to depend on the half-angle of the taper. Over a range of small angles, the analytical model was found to accurately predict this behaviour.

1. Introduction

The transmission of stress waves within granular medium is a complicated issue because of the heterogeneity and nonlinearity inherent in these systems. In the last decades, propagation of stress waves in granular media has been extensively studied and reported in the literature. These treatments range from analytical methods (Gregor and Rumpf, 1975; Ocone and Astarita, 1995; Weir, 2001), to experimental investigation (Liu and Nagel, 1992; Ben-Dor et al., 1997; Hardin and Richart, 1963; Musmarra et al., 1995; Tournat et al., 2004) to more recent studies using numerical simulation (Melin, 1994; Berezin et al., 2001; Hostler and Brennen, 2005; Oveisy et al., 2009; Mouraille et al., 2009).

The study of stress waves in granular medium is of general interest in many industrial applications. An understanding of the propagation of stress waves in granular medium is required for the study of the dynamic aspects of industrial handling of bulk materials. For example, the discharge of granular materials from silos is often characterised by vibrations or pulsations known as 'silo quaking' (Roberts and Wensrich, 2002). Wensrich (2002) proposed that these pulsations or vibrations are due to compression and rarefaction waves in the granular materials, which are caused by slip-stick motion between the granular material and the silo walls.

Borzsonyi and Kovacs (2011) reported experimental observation of sound waves in a granular material during a resonant test-scale silo discharge. The grain motion was tracked by high-speed imaging while the resonance of the silo was detected by piezo-electric accelerometers and acoustic methods. Their major observation was that the wave velocity was not constant throughout the silo but increased considerably toward the transition zone in the lower end of the silo. In this region, the strong stress oscillations were observed. Moreover, the amplitude of the oscillations increased with height and led to slip-stick motion in the upper part of the silo. This conclusion supports the viewpoint of Roberts and Wensrich (2002) and Wensrich (2002).

Propagation of pressure fluctuations such as these were first studied analytically by Boutreux et al. (1997), who proposed a dynamic version of the Janssen model in a column of linear elastic material in the absence of gravity. This contribution studied the role of wall friction on the behaviour of pressure steps in the system (analogous to the water hammer problem). Friction was mobilised in a direction opposite to the direction of propagation, and it was shown that in this situation, a compression front decayed exponentially.

Using a similar dynamic version of Janssen's silo model, Wensrich (2002) later showed that the action of wall friction also has an important role in the growth of rarefaction waves. In the model, it was assumed that friction was fully mobilised in the upward direction (akin to Janssen's original model) and the action of a rarefaction wave was to overcome friction and motivate the flow of initially stationary material. Using this model it was shown analytically that a rarefaction wave starting at the base of the silo, initially of magnitude σ_0^+, grows with the following form:

$$\sigma^+(x,t) = \sigma_0^+ e^{(x/2X_0)} \tag{1}$$

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in which x is the distance travelled by the wave (in the upward direction), σ^+ is the stress disturbance in the axial (x) direction, $X_0 = D/4\mu k$ is the Janssen characteristic depth of the column, D is the diameter of the column, μ is the coefficient of friction between the material and the column walls and k is the coefficient of lateral pressure.

In this paper, we will extend the work of Wensrich (2002) to the study of tapered column of granular material. In particular, we will examine the geometry effect on the rate of growth of rarefaction waves. Propagation of elastic waves in solid body with variable cross section has been extensively studied and reported in the literature (Brillhart and Dally, 1968; Yang and Hassett, 1972; Rogge, 1971; Kenner and Goldsmith, 1968). Kenner and Goldsmith (1968) and Lewis et al. (1969) experimentally studied the response of conical bars to impact loads. Suh (1968) determined the stress pulse amplification ratio in truncated cones for various cones angles. Rogge (1971) developed, by perturbation analysis, an equation for elastic wave propagation in an arbitrary solid of revolution.

Our approach will adopt the analytical technique presented by Wensrich (2002). In addition to developing a new description for tapered columns with frictional boundaries, we will also present the results of a finite element (FE) study of the same phenomenon using the uncoupled Arbitrary Lagrangian–Eulerian formulation (ALE). The numerical results will serve to verify the predictions of the analytical models and provide a method to examine the limits of the validity of the modelling assumptions.

2. Convergent and divergent columns

2.1. Physical system and modelling assumptions

To study the geometry effect on wave propagation in granular solid, we will develop two models: a converging axisymmetric column (conical with apex downwards; see Fig. 1); and a diverging axisymmetric column (apex upwards). To facilitate analytical solutions, we make the assumption that the half angle of the cone is small in each case. Principally for this reason, we will refer to these systems as converging/diverging columns, rather than using the term "hopper" for the converging case. The base of the column consists of a boundary that moves as a function of time in order to disturb the system—creating waves that travel up through the material.

In each case, we assume that the cone of material is acted upon by frictional forces that originate from the interaction of the material with the walls of the container as is typical in the analysis of a conical hopper (Nedderman, 1992). We assume that Coulomb friction at the wall interface is fully mobilised in the upward direction with a coefficient of friction of μ. Friction is a difficult phenomenon to model analytically due to its incremental non-linearity. For this reason we restrict our analysis to the case where the disturbance creates rarefaction waves in the material and friction is maintained at its maximum limiting value.

The assumption that the cone angle is small allows us to assume the material moves in one direction only, and that the velocity profile is approximately constant across the diameter-with the exception of a small boundary layer. We will also make the assumption that the disturbances are small and the density does not vary significantly, and that the transverse principal stress is in a constant ratio to the axial stress σ_x.

From this perspective, we can restrict the number of unknown state variables in this system to two: the velocity and the principal stress in the axial direction emanating from the apex of the cone. Both of these variables are functions of both time and position.

2.2. Converging column model

Consider a converging column of half-angle α as shown in Fig. 2. To compare this model directly to the previous work by Wensrich (2003), we consider the case where the conical column is truncated at the base with outlet diameter D and the coordinate x measured from the base. The two coordinates are related by

$$x = r - h, \text{ or, } = r - \frac{D}{2\sin\alpha} \quad (2)$$

The forces acting on a differential element of thickness dx and surface area S are the forces due to gravity, wall boundary stresses and the internal stresses as shown in Fig. 2. Summing the forces in the vertical direction, the equation of motion for this thin shell is

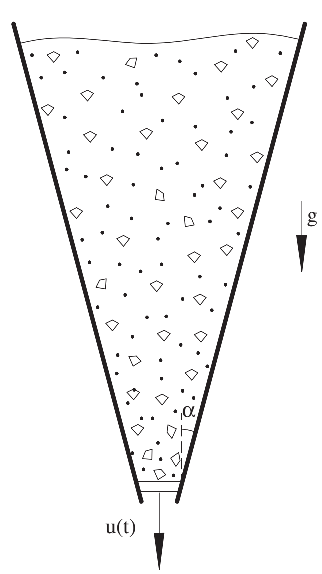

Fig. 1. A cone of granular material.

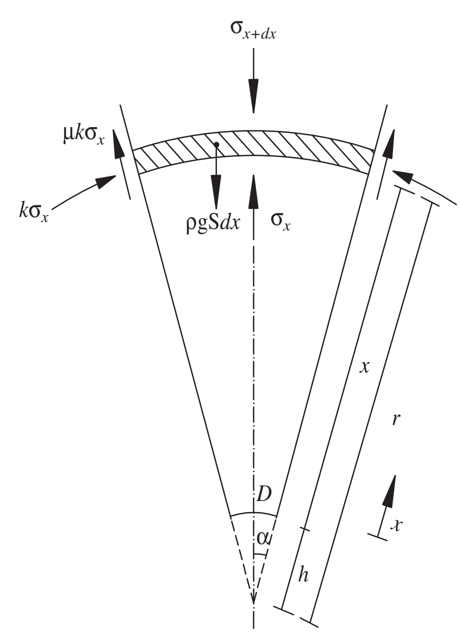

Fig. 2. Forces acting on a differential element of material in the converging column.