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Numerical simulations of crack propagation in screws with phase-field modeling

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ABSTRACT

In this work, we consider a phase-field framework for crack propagation problems in elasticity and elasto-plasticity. We propose a rate-dependent formulation for solving the elasto-plastic problem. An irreversibility constraint for crack evolution avoids non-physical healing of the crack. The resulting coupled two-field problem is solved in a decoupled fashion within an augmented Lagrangian approach, where the latter technique treats the crack irreversibility constraint. The setting is quasi-static and an incremental formulation is considered for temporal discretization. Spatial discretized is based on a Galerkin finite element method. Both subproblems of the two-field problem are nonlinear and are solved with a robust Newton method in which the Jacobian is built in terms of analytically derived derivatives. Our algorithmic developments are demonstrated with several numerical tests that are motivated by experiments that study failure of screws under loading. Therefore, these tests are useful in practice and of high relevance in mechanical engineering. The geometry and material parameters correspond to realistic measurements. Our goal is a comparison of the final crack pattern in simulation and experiment.

1. Introduction

Presently, crack propagation in solids is one of the major research topics in energy, environmental, and mechanical engineering. Consequently, many models and numerical techniques have been investigated to date. Specifically, Griffith's model [1] for quasi-static fracture evolution has been successfully applied. Here, the crack propagates if the rate of elastic energy dissipation per unit surface area of the increment step is equal to a critical energy release rate G_c; if it is less, the crack does not move. On the contrary, it is unstable if G_c exceeds the critical value. Griffith found that G_c is related to the crack surface energy increase. In the case of isotropic linear materials, the critical energy release rate is linked to the stress intensity factor.

The solution of crack representation and propagation requires special techniques for their numerical treatment. In recent years, different approaches have been proposed such as the extended finite element method (XFEM) by [2] and generalized finite elements (GFEM) methods [3] both based on the partition of unity method of [4] in which the displacement field is enriched with discontinuities. In the last decade, variational and phase-field techniques for crack propagation in solid mechanics have gained increased interest. The original model has been proposed by Francfort and Marigo [5]. Numerical simulations and mathematical investigation have been subject, e.g., in [6,7]. An important extension towards a thermodynamically-consistent phase-field formulation has been suggested by Miehe et al. [8,9] and further simulations and numerical extensions (also by other researchers) are found in [10–12]. Specifically, a finite element framework for multiphysics phase-field simulations has been presented in [13]. Instead of modeling the discontinuities explicitly (like in XFEM and GFEM), employing phase-field, the lower-dimensional crack surface is approximated with an auxiliary function φ with values 0 in the crack and 1 in the undamaged material; an illustration by means of a numerical simulation is provided in Fig. 2. Furthermore, φ smoothly interpolates between 0 and 1 leading to a diffusive transition zone that is of size ε, a so-called length-scale model regularization parameter. From a computational point of view, the phase-field approach is attractive since crack nucleation, length increment, and the path are automatically

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included in the model [7]. On the other hand, the diffusive transition zone tends to smear out the sharp crack surface and the characteristic length-scale parameter ε must be chosen accordingly. Here, an adaptive local grid refinement technique can be used to increase crack surface resolution while keeping the computational cost low. Additionally, some of the challenges associated with a non-convex energy functional require careful developments as outlined and addressed in [7,14,15].

While it is fairly well understood how phase-field fracture models act in brittle materials with elasticity, only a few references exist to date on ductile crack propagation (e.g., [16–19]). Here, the last reference focuses on numerical tests of I-shaped specimen that have some similarities to screw simulations as we have them in mind. From phase-field modeling point of view our proposed approach for fracture propagation in elasto-plastic materials has similarities with the technique described in [18] in which quasi-static brittle fracture propagation in elasto-plastic solids has been formulated.

Our first aim in this paper is an algorithmic extension of elasto-plastic phase-field fracture combined with an augmented Lagrangian formulation for treating the crack irreversibility [12]. This approach is modeled in terms of a rate-dependent formulation [20] in which continuous time derivatives are approximated with a backward in time difference quotient, i.e.,

$$\partial_t \varphi \approx \varphi - \varphi^{m-1} \leqslant 0.$$

Here, φ denotes the phase-field fracture variable and φ^{m-1} its state at the previous time step t^{m-1}. Spatial discretization is done with finite elements, which are well-known in solid mechanics. Due to the crack irreversibility and the elasto-plastic material behavior the resulting scheme has two types of nonlinearities for which a robust Newton solver is employed. The capabilities of our approach are substantiated with a benchmark example. Several screw tests constitute the main goal that include comparisons with crack growth in elasticity and elasto-plasticity as well as comparative studies of crack pattern in experiments and finite element simulations. The latter aspect has presently high relevance in mechanical engineering.

In summary, the importance of our present paper is twofold:

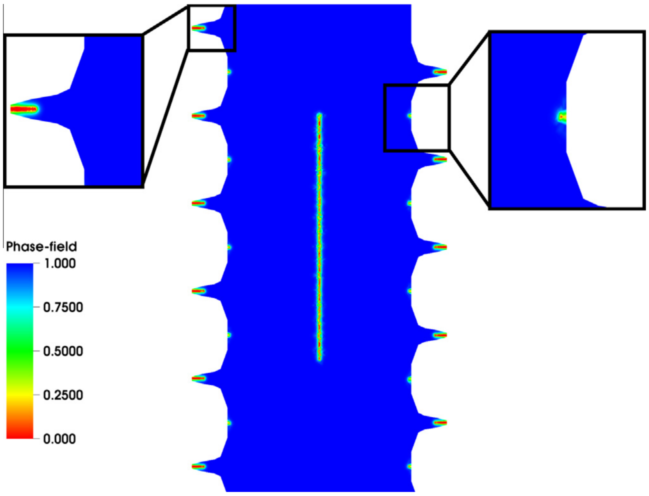

Fig. 2. Initialization of cracks modeled with phase-field. Crack patterns are described with an indicator function φ (i.e., the phase-field function) with 0 values in the crack (red color) and 1 (blue color) in the undamaged zone. The diffusive transition zone has width ε (green/yellow) and here the phase-field function has values that smoothly vary between 0 and 1. This color scale is also employed in Section 4 to visualize cracks and damage in terms of the phase-field variable. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

- Formulating elasto-plastic crack propagation with a thermodynamically consistent phase-field model and its rate-dependent formulation within a robust augmented Lagrangian solution algorithm.
- Investigation of material damages that are presently of high importance in mechanical engineering for learning reasons for material damage and failure. Fig. 1 shows some characteristics of material damage in screws. Typical material damages are closed wrinkles in the middle of thread flanks, seams, cracks and hollow-rolled screws. The cause of material damage is often a wrong set up of the rolling machine. We note that in the production process, the thread of the final screw is obtained from rolling of an initially un-machined blank. Consequently, numerical simulations (here with phase-field; see Fig. 2) greatly help to better understand the mechanisms of failure.

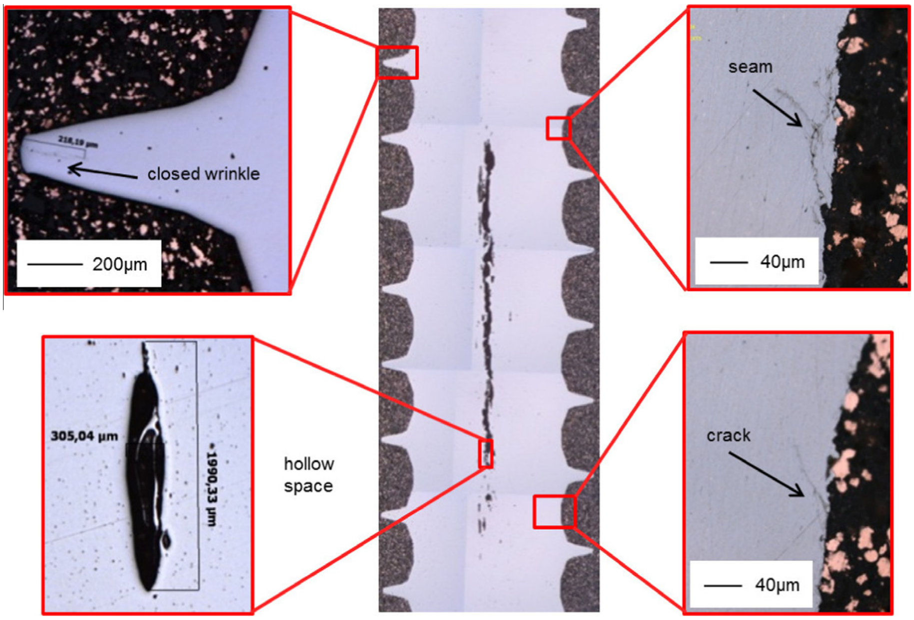

Fig. 1. Illustration and characteristic length scales for material damage in screws.