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Effect of Dzyaloshinsky–Moriya interaction on magnetic vortex: A real-space phase-field study

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ABSTRACT

A real-space phase field model is developed to investigate the effect of Dzyaloshinsky–Moriya (DM) interaction on magnetic vortex in ultrathin ferromagnetic film. Based on the time-dependent Ginzburg–Landau (TDGL) equation, the phase field model takes into account the DM interaction of magnetizations, which includes the coupling between the magnetization and magnetization gradient. The governing equations in the phase field model are solved simultaneously by means of a nonlinear finite elements method, which can be employed to simulate magnetization vortex without periodic boundary condition. The simulation results demonstrate that the DM interaction has significant influence on the structure of the magnetization vortex. It is found that the DM interaction induces an out-of-plane magnetization on the edge of the vortex and enlarges the size of the vortex core. The magnitude of the out-of-plane magnetization at the vortex core and the edge increases with the increase of DM constant. Besides, the handedness of magnetization vortex also changes when the sign of DM constant changes.

1. Introduction

Based on its unique structure, the magnetization vortex in a nanoscale magnet is defined by the polarity and circulation. Specifically, the magnetization vortex is characterized by nanometer sized central region with the magnetization perpendicular to the plane ($p = \pm 1$, up or down) and an in-plane curling magnetization ($c = \pm 1$, counterclockwise or clockwise) [1,2]. It has attracted numerous attentions during last decades, especially after the finding that the vortex polarity could be manipulated by the in-plane electrical current [3,4]. The switching property of magnetization vortex can be exploited for a new kind of memory devices in practice. In the literature, various experimental and theoretical methods have been developed to study dynamic properties of magnetization vortex. Among them, many advanced instruments, such as the magnetic force microscopy [5] and time-resolved magnetic X-ray microscopy [6], are employed to provide a direct observation on the magnetization vortex. Meanwhile, the micromagnetic model [7,8] and phase-field model [9] are developed to elucidate the underlying mechanism of evolution process of magnetization vortex.

Due to magnetostrictive effect, the stress field can induce the change of magnetization state at room temperature. However, because of the complicated elastic solutions associated with the arbitrary domain structures in ferromagnetic materials, most of the micromagnetic models have not taken into account the effect of inhomogeneous stress although Shu et al. have obtained the inhomogeneous stress via a modified boundary integral formalism for two-dimensional cases [10]. To model the temporal evolution of three-dimensional domain structures of giant magnetostrictive materials, a phase field model that involves the coupling between magnetization and strain is proposed by Zhang and Chen [11]. However, the periodic boundary conditions are employed in their calculations, in which geometry and boundary conditions of the simulated model are limited to some extent. In our previous work, a real space phase-field model which includes the magneto-elastic coupling is developed [9]. By means of nonlinear finite element method, the model can be applied to simulate the domain evolution of ferromagnetic materials with arbitrary boundary conditions and geometries. As demonstrated in the previous work, the model successfully predicted the evolution of multi-vortices in ferromagnetic nano-platelets subjected to a normal stress [12], and the polarity switching of magnetization vortex in ferromagnetic circular nanodots by a shear strain [13]. In both cases, there are free boundaries.

When free boundaries appear, a novel intrinsic Dzyaloshinsky–Moriya (DM) interaction that arises from the spin–orbit coupling due to the lack of inversion symmetry at surfaces has been found to play a significant role in the formation of magnetic vortex

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[14–16]. For example, Im et al. demonstrated that the DM interaction is decisive for the asymmetric formation of vortex states [2]. Luo et al. showed that DM interaction would influence the size of vortex core and induce an out-of-plane magnetization at edge of the disk [17]. In the present work, we extend previous phase-field model of ferromagnetic materials [12] by taking into account the DM interaction, in which the coupling between the magnetization and magnetization gradient is included. The magnetization distributions of the ferromagnetic vortex in square thin films are simulated with various DM constants.

2. Phase-field method

In the phase-field model of ferromagnetic materials, a vector field $M(\boldsymbol{r},t)$ is used as the order parameter to describe the spatial distribution and temporal evolution of magnetization, where $\boldsymbol{r} = (x_1, x_2, x_3)$ represents the local position vector and t is time. For magnetic materials, the DM energy can be written as $\omega_D = D(\Lambda^i_{kj} + \Lambda^j_{ik} + \Lambda^k_{ji})$, in which D is the DM constant and Λ^k_{ij} is the so-called Lifshitz invariants [18]. The Lifshitz invariants $\Lambda^k_{ij} = m_i\partial_k m_j - m_j\partial_k m_i$ denotes the coupling between the magnetization and magnetization gradient [15,19,20], in which $\partial_k m_i \equiv \partial m_i/\partial x_k$ are the spatial derivatives of the component of magnetization unit vector m_i with respect to x_k. The magnetization unit vector is defined as $\boldsymbol{m} = \boldsymbol{M}/M_s$ and M_s is the magnitude of saturation magnetization. The DM energy favors spatially chiral states where the magnetization rotates with a fixed turning sense in the plane perpendicular to the propagation direction. The DM energy can be expressed as

$$\omega_D = \frac{D}{M_S^2}(M_3M_{2,1} - M_2M_{3,1} + M_1M_{3,2} - M_3M_{1,2} + M_2M_{1,3} - M_1M_{2,3}) \tag{1}$$

In general, there are three possible orientations for the symmetry axis of uniaxial magnetic material. For each orientation, only part of the energy expression exits in Eq. (1). In order to distinguish the DM energy for different orientations of symmetry axis, the total DM energy in Eq. (1) can be divided into three parts as follows,

$$\begin{aligned}
\omega_{D1} &= \frac{D_1}{M_S^2}(M_1M_{3,2} - M_3M_{1,2} + M_2M_{1,3} - M_1M_{2,3}),\\
\omega_{D2} &= \frac{D_2}{M_S^2}(M_3M_{2,1} - M_2M_{3,1} + M_2M_{1,3} - M_1M_{2,3}),\\
\omega_{D3} &= \frac{D_3}{M_S^2}(M_3M_{2,1} - M_2M_{3,1} + M_1M_{3,2} - M_3M_{1,2}),\\
\omega_D &= \omega_{D1} + \omega_{D2} + \omega_{D3},
\end{aligned} \tag{2}$$

where ω_{D1}, ω_{D2} and ω_{D3} represent the DM energies when the symmetry axis parallel to the x_1, x_2 and x_3 directions, respectively. The DM constants in Eq. (2) can be expressed as $D_1 = D_2 = D_3 = D/2$. In addition to the DM energy, other energy terms should also be included in the phase field model. When the temperature is below the Curie point, the total free energy density of the magnetic material can be given by

$$E = E_{ani} + E_{gra} + E_{pur} + E_{cou} + E_{mag} + E_{con} + \omega_D \tag{3}$$

where E_{ani}, E_{gra}, E_{pure}, E_{coup}, E_{mag} and E_{con} are the magneto-crystalline anisotropy energy, gradient energy, pure elastic energy, magneto-elastic coupling energy, magnetic energy, constraint energy, respectively. The total free energy of a cubic ferromagnetic material can be expressed as

$$\begin{aligned}
E(m_i, m_{i,j}, \varepsilon_{ij}, H_i) =& \frac{K_1}{M_s^4}\left(M_1^2M_2^2 + M_1^2M_3^2 + M_2^2M_3^2\right) + \frac{K_2}{M_s^6}M_1^2M_2^2M_3^2\\
&+ \frac{A_{ex}}{M_s^2}\left(M_{1,1}^2 + M_{1,2}^2 + M_{1,3}^2 + M_{2,1}^2 + M_{2,2}^2 + M_{2,3}^2\right.\\
&\left.+ M_{3,1}^2 + M_{3,2}^2 + M_{3,3}^2\right) + \frac{1}{2}c_{11}\left(\varepsilon_{11}^2 + \varepsilon_{22}^2 + \varepsilon_{33}^2\right)\\
&+ c_{12}(\varepsilon_{11}\varepsilon_{22} + \varepsilon_{11}\varepsilon_{33} + \varepsilon_{22}\varepsilon_{33}) + 2c_{44}\left(\varepsilon_{12}^2 + \varepsilon_{13}^2 + \varepsilon_{23}^2\right)\\
&- \frac{3\lambda_{100}}{2M_S^2}(c_{11} - c_{12})\left(\varepsilon_{11}M_1^2 + \varepsilon_{22}M_2^2 + \varepsilon_{33}M_3^2\right)\\
&- \frac{6\lambda_{111}}{M_S^2}c_{44}(\varepsilon_{12}M_1M_2 + \varepsilon_{13}M_1M_3 + \varepsilon_{23}M_2M_3)\\
&- \frac{1}{2}\mu_0\left(H_1^2 + H_2^2 + H_3^2\right) - \mu_0(H_1M_1 + H_2M_2 + H_3M_3)\\
&+ A_s(M - M_s)^2 + \frac{D_1}{M_S^2}(M_1M_{3,2} - M_3M_{1,2} + M_2M_{1,3} - M_1M_{2,3})\\
&+ \frac{D_2}{M_S^2}(M_3M_{2,1} - M_2M_{3,1} + M_2M_{1,3} - M_1M_{2,3})\\
&+ \frac{D_3}{M_S^2}(M_3M_{2,1} - M_2M_{3,1} + M_1M_{3,2} - M_3M_{1,2}).
\end{aligned} \tag{4}$$

In which $M = \sqrt{M_1^2 + M_1^2 + M_3^2}$ is the magnitude of magnetization vector and A_s is the constraint energy constant. In the magnetocrystalline anisotropy energy, K_i ($i = 1, 2$) and M_s are the magnetocrystalline anisotropy constants and the magnitude of saturation magnetization, respectively. The exchange energy is induced by the spatially inhomogeneous magnetization, where A_{ex} is the exchange stiffness constant. The coupling energy is expressed by strain and magnetization, in which λ_{100} and λ_{111} are the magnetostrictive constants. For the cubic material, there are three elastic constants, namely, c_{11}, c_{12}, and c_{44}. The magnetic energy includes both the external magnetic field energy (if any) and demagnetization energy, in which μ_0 is the permeability of a vacuum.

The temporal evolution of the magnetization distribution in the materials is described by the time-dependent Ginzburg–Landau (TDGL) equation [21,22]

$$\frac{\partial \boldsymbol{M}(\boldsymbol{r},t)}{\partial t} = -L\frac{\delta F}{\delta \boldsymbol{M}(\boldsymbol{r},t)} \tag{5}$$

where L is the kinetic coefficient, $F = \int_V E dV$ is the total energy of the ferromagnetic materials. For overdamping cases, the TDGL equation is not only the simplest continuum model for magnetization evolution, but also gives the same evolution process as the LLG equation. It has been employed to study the domain evolution of ferromagnetic materials [21]. In the present study, the evolution speed is slow and the overdamping condition is valid. For simplicity, the TDGL equation is employed, which can be expressed in the form of partial differential equation as

$$\frac{\partial M_i}{\partial t} = -L\left[\frac{\partial E}{\partial M_i} - \frac{\partial}{\partial x_j}\left(\frac{\partial E}{\partial M_{i,j}}\right)\right] \tag{6}$$

Besides, the mechanical equilibrium equation $\frac{\partial}{\partial x_j}\left(\frac{\partial E}{\partial \varepsilon_{ij}}\right) = 0$ that ignores the body force, and the Maxwell's equation $\frac{\partial}{\partial x_j}\left(-\frac{\partial E}{\partial H_i}\right) = 0$, are introduced to describe the evolution of elastic strain and magnetic field. A 3D nonlinear finite element method is applied to solve above three governing equations.

To solve these above partial differential equations with finite element method, the weak form of the governing equations can be expressed as

$$\begin{aligned}
&\int_V \left[\frac{\partial E}{\partial \varepsilon_{ij}}\delta\varepsilon_{ij} + \frac{\partial E}{\partial H_i}\delta H_i + \left(\frac{\partial M_i}{\partial t} + L\frac{\partial E}{\partial M_i}\right)\delta M_i + L\frac{\partial E}{\partial M_{i,j}}\delta M_{i,j}\right]dV\\
&\quad= \int_S \left[t_i\delta u_i - B_n\delta\varphi + \left(\frac{\partial E}{\partial M_{i,j}}n_j\right)\delta M_i\right]dS
\end{aligned} \tag{7}$$