Computational Materials Science 102 (2015) 27–44

Contents lists available at ScienceDirect

Computational Materials Science

journal homepage: www.elsevier.com/locate/commatsci

Adaptive thermo-mechanical finite element formulation based on goal-oriented error estimation ☆

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ARTICLE INFO

Article history:
Received 13 March 2014
Received in revised form 14 January 2015
Accepted 16 January 2015
Available online 28 February 2015

Keywords:
Thermo-mechanical
Thermoelasticity
Recovery error estimation
SPR
WSPR
L_2-PR
Adaptive refinement
Mesh adaptivity
Quantity of interest
Goal-oriented

ABSTRACT

In this paper, a *rh*-adaptive thermo-mechanical formulation based on goal-oriented error estimation is proposed. The goal-oriented error estimation relies on different recovery-based error estimators, i.e. the superconvergent patch recovery (SPR), L_2-projection patch recovery (L_2-PR) and weighted superconvergent patch recovery (WSPR). A new adaptive refinement strategy (ARS) is presented that minimizes the error in a quantity of interest and refines the discretization such that the error is equally distributed on the refined mesh. The method is validated by numerous numerical examples where an analytical solution or reference solution is available.

1. Introduction

In recent years, substantial attention has been devoted to thermoelastic problems. The thermoelastic governing equations can be formulated coupled or uncoupled. In the theory of uncoupled thermoelasticity, the temperature and heat distribution are described by a parabolic partial differential equation which does not contain any strain or displacement terms. The uncoupled theory of thermoelasticity has two unrealistic assumptions: 1. The displacement field does not affect the temperature field and 2. it yields an infinite wave propagation speed. These two shortcomings have been overcome by the coupled theories of thermoelasticity [1–3], that account for wave propagation effects.

In numerical analysis, one concern after solving the problem is the accuracy of the approximation. Errors in the results are inevitable, so different error estimation techniques have been developed to evaluate and control them. Generally, error estimators can be classified as *a priori* or *a posteriori* methods. In this paper *a posteriori* error estimation is considered.

Babuška and Rheinboldt were one of the first to investigate *a posteriori* error estimation for the finite element (FE) method. Their paper on finite element error estimation for two point elliptic boundary value problem in 1978 [4] became the basis for adaptive mesh refinement [5–18]. A posteriori error estimators can be classified into residual-based [19–22] and recovered-based.

Recovery based error estimations are based on so-called recovered gradient fields which are more accurate than the original finite element solution. Several procedures have been proposed to calculate the recovered gradient fields. In 1971, Hinton and Campbell [23] suggested a procedure to calculate the stresses at all nodes by extrapolating the Gauss point values. Another recovery technique, which assumes that the recovered stresses in the element are interpolated in the same manner as the displacements, was proposed by Brauchli and Oden [24]. This method is known as L_2-projection (L_2-PR) method and calculates the recovered stress field by a global least square fitting of the stresses. In 1987, a simple recovery based error estimator was introduced by Zienkiewicz and Zhu [25]. It was based on comparing smoothed solution gradients with the gradients of the solution themselves. This technique became known as $Z-Z$ or Z^2 error estimation. The main advantages of the Z^2 error estimator are its simple implementation and

☆ Special Issue Paper: Polymeric Composites.

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http://dx.doi.org/10.1016/j.commatsci.2015.01.044
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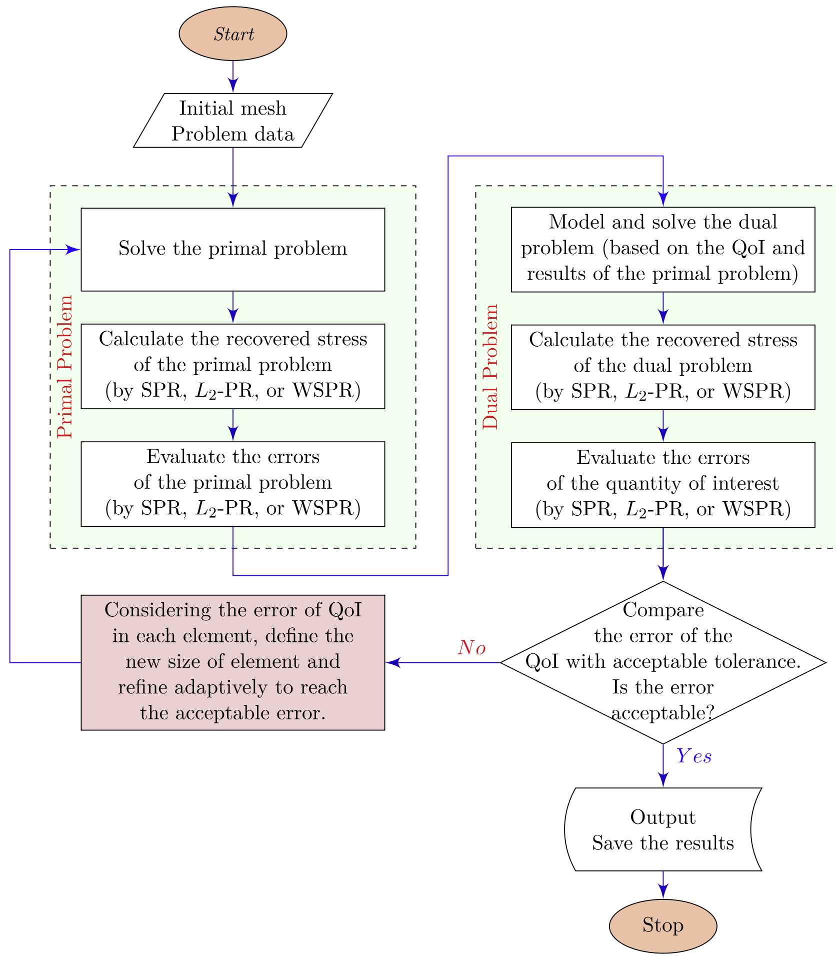

Fig. 1. Program algorithm of current study.

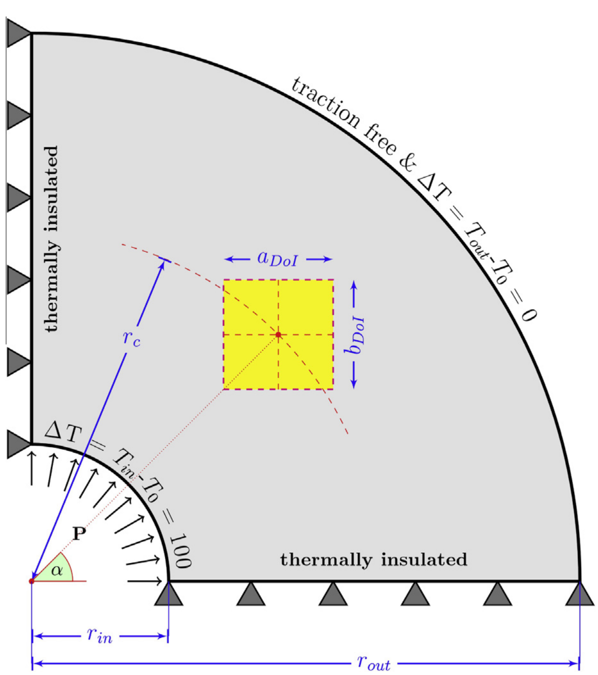

Fig. 2. Geometry and boundary conditions of model of thick-wall cylinder subjected to internal pressure and temperature change (example Section 6.1).

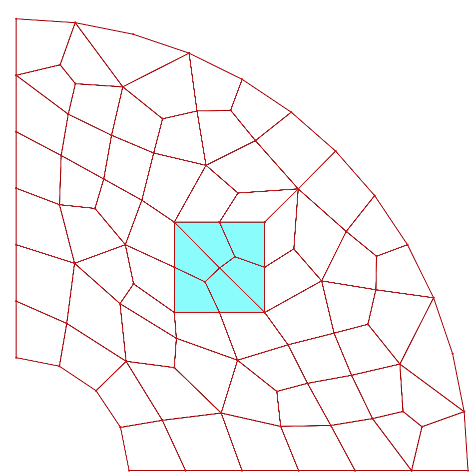

Fig. 3. Initial mesh used to analyze the benchmark problem (Section 6.1).

reasonable efficiency. Later, the accuracy of the method was improved by modifying the recovery technique and introducing superconvergent patch recovery (SPR) method, see [26,27].

Several improvements of the conventional SPR method have been proposed. In 1994, Wiberg et al. introduced the SPR-EB as an extension of SPR technique [28] which was also independently