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Particle motion and separation in a laminar tube flow with downstream enlargement

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ABSTRACT

Particle classification becomes difficult when the difference in density between particle and fluid is low or negligible and the fluid is viscous. For such applications, a process capable of separating the particles according to their size is needed. Such applications are, e.g. found in biological systems for cell separation or in the removal of gel particles from polymer melts. Particle transport in laminar tube flows at low but non zero Reynolds numbers leads to accumulation of large particles near the tube center and forms a particle free zone near the wall. Small particles find their position on their equilibrium radius. Downstream widening of the flow enhances segregation between large and small particles. Large particles can be collected in a centered collector tube downstream, whereas small particles follow their streamlines around the collector tube and can be removed with the remaining flow. The said particle migration is observed when the ratio of particle to tube diameter is $0.2 < d/D < 0.51$ and the tube Reynolds number is in between $0.2 < Re < 40$. CFD simulations reveal the shape of the streamlines in the downstream enlargement with different tube Reynolds number. The efficiency of the classification process is characterized. Particles need a sufficient transportation length in the tube for proper demixing. This effect is analyzed by a laser sheet illuminated system within an acrylic glass tube.

1. Introduction

Traditional size classification processes apply sieves or sedimentation. Particle separation with sieves is feasible in a low viscous fluid environment under repeated contact with the meshes to provide the necessary probability of transition. This separation method becomes difficult when the viscosity is high and the strong interaction between particles, fluid and mesh impairs efficient segregation. Sedimentation is also an alternative for separation according to the particle size due to different settling velocities of large and small particles in the fluid. This process is applied for example in hydrocyclones for particle separation at cut off size in the range of $5 < d < 250\ \mu\text{m}$ (Schubert, 2003). A low fluid viscosity facilitates the separation. However, sedimentation is only feasible at sufficiently large density differences between particles and fluid.

The axial movement of particles in cylindrical ducts at low duct and particle Reynolds numbers was extensively analytically analyzed recently (Bhattacharya et al., 2010). However, the radial movement of particles was not yet considered. In a duct with circular cross section diameter D, particles with diameters $d \ll D$ in the range $r > r'$ are forced towards the tube axis because of the flow constraints close to the channel wall (see Fig. 1). Particles in the range $r < r'$ are deflected towards the wall due to the non-uniform shear strain on the particle surface due to the asymmetrical velocity field. As a result, particles are collected at the equilibrium radius r'. Larger particles $d > 0.1D$ separate radially and the dispersed particles form ring zones (Lou and Pan, 2003). The position of the equilibrium radius then depends on the ratio d/D. Therefore different particle tracks can be expected allowing for particle separation, e.g. by concentric separation tubes dividing the flow region into different channels after leaving the tube.

This behavior had been extensively examined in numerous publications. Segré and Silberberg (1961) performed early experiments in a vertical tube with small particles in the size range of $0.07 < d/D < 0.15$. The particle motion within a tube Reynolds number between 3.2 and 173 were examined at different flow rates. The particles were concentrating in an annular zone. The radius of the zone was determined as $r'/R \approx 0.6$ (Segré and Silberberg, 1961, 1962, 1962a). Bauckhage examined the influence of the particle size on the equilibrium radius. The maximum possible displacement for a particle is given by

$$r_{\max} = R - \frac{d}{2} \tag{1}$$

R is the tube radius and d is the particle diameter. A dependency of the equilibrium radius r' on the tube Reynolds number within

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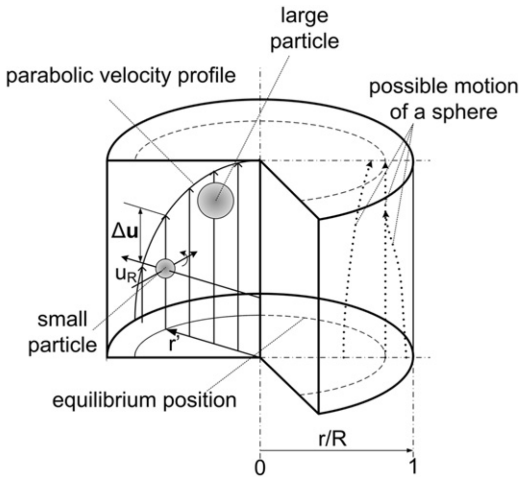

Fig. 1. Particle motion in a laminar tube flow with parabolic velocity profile.

the range $36 < Re < 1000$ was found and described by Bauckhage (1973, 1974, 1975) as

$$\frac{r'}{r_{\max}} = \frac{1}{\sqrt{2}} + 0.064 \ln \frac{Re}{36} \tag{2}$$

and for $1 < Re < 36$ as

$$\frac{r'}{R} = \left(R - \frac{d}{D}\right)\frac{1}{\sqrt{2}} \tag{3}$$

The tube Reynolds number is defined as $Re = (\rho u D)/\mu$ and the particle Reynolds number is defined as $Re_P = (\rho \Delta u d)/\mu$; ρ is the density of the fluid and μ is the viscosity of the fluid.

Bauckhage also obtained an analytical description for the path length in the tube, which is needed to position the particles on their equilibrium radii. It was obtained as

$$L' = \frac{l'}{D} = 1.43\, Re^{-0.5} \left(\frac{2R}{d}\right)^2 \left(9.2 \frac{R}{r'} - 3.6 \frac{r'}{R}\right) \tag{4}$$

Eq. (2) predicts smaller equilibrium radii for larger particles and must be zero for particles $d/D = 1$. Large particles must pass the tube more or less with their centers close to the centerline of the duct. In the case of high particle concentrations, small particles also accumulate at the tube center but the particle free zone close to the wall still exists. Further experiments were performed by Saffman (1956), Rubin (1977) and Matas et al. (2004). Buggisch and Muckenfuss (2002) investigated the influence of solid boundary walls in shear flows of suspensions and the induced particle transport (Muckenfuss and Buggisch, 2003; Muckenfuss, 2006). They developed a model describing effects such as the occurrence of pseudo wall slip or the discontinuities of the particle concentration. No detailed application of the size dependent particle migration on particle separation was studied.

Klein (1999) separated particles according to their size in laminar flow within micro channels. The separation effect was intensified by additional forces, caused by gravitation, temperature gradients or a cross flow. Small particles diffused to the center of the flow with higher flow velocities and large particles were forced towards the wall and moved with lower flow velocity. Yamada and Nakashima (2004) describe a process for particle separation in a laminar flow with a side stream to the tube (Takagi and Yamada, 2005). The side stream deflects the particles against the opposite wall of the tube. Small particles move a wider distance towards the opposite wall compared to the larger ones hampered by the tube wall. The particles were then separated perpendicular to the main flow direction at the channel outlet according to their size. A downstream enlargement improved the separation in the micro channel. Similar separation processes were presented by Pamme (2007) and Huang and Cox (2004). Particle separation without side stream addition was not examined in their paper.

In order to apply migration behavior to particle separation in laminar duct flow, the intention was to analyze a separation method for particles with negligible density difference between particles and fluid. Dilution by side flow addition should be avoided, on the contrary, the tube outflow is forced into a laminar source flow giving space for large separation cross-sections downstream of the outlet with low plugging tendency. The process as proposed can perform the separation in one or more separation stages. Tube Reynolds number $0.2 < Re < 40$ were considered with negligible density differences between particle and flow as also reported in Walzel et al. (2007) and Matulka and Walzel (2010). Meanwhile it is evident that the deflection of particles from the tube wall and their migration from the tube center to the equilibrium radius is based on inertia effects (Di Carlo 2009). For small Reynolds numbers the particles move very slowly towards their equilibrium radius and the path length for positioning on the equilibrium radius becomes very long. For $Re < 0.1$ the particle positioning on their equilibrium radius practically vanishes.

The movement of a larger number of poly-disperse particles in laminar flow is superimposed by particle interaction in a stochastic manner. The causes are different starting conditions of the particles and particle–particle interactions due to different axial velocities at different equilibrium radii.

2. Experimental setup

The experimental setup, as shown in Fig. 2, consists of a vertical acrylic glass tube with an internal diameter of $D = 6$ mm and a total length of 2000 mm. The tube extends into a chamber with a rectangular side length of 200×200 mm^2. The chamber is used as a downstream enlargement for the flow. The level height of the liquid in the chamber is 110 mm. At a variable distance between 8 and 38 mm from the tube outlet a collector tube with an inner diameter of 16 mm is fixed to provide a large cross-sectional area. Two gear pumps convey the fluid in a recycle loop while two filters were installed to protect the pumps from the injected particles. The experimental liquid is collected in a main tank. At a distance of 100 mm from the entry into the vertical tube, a horizontal 6 mm diameter tube is connected laterally for particle injection (particle injection 1). This horizontal tube can be removed and filled with a suspension prepared in advance. To avoid any reflux to the injector, a sphere valve is installed. The radially adjustable tube outlet in the funnel-shaped expansion sets the exact starting position for the particles (particle injection 2). With this device, the transportation length required by the particles to find their equilibrium radii as a function of the particle starting position was investigated.

The experimental liquids were mixtures of glycerol and water, which allow variation of the dynamic viscosity and the density of the mixtures. In the present approach, mixtures of 86–96 wt% glycerol were used to obtain viscosities within the range of $0.020 < \mu < 0.334$ Pa s. It was important to find particles with a density close to that of the fluid. The particles were formed by dripping a 2 wt% of alginate, 1 wt% powder cellulose and 0.05–0.15 wt% of micro-balloons into a 0.3 M CaCl-solution from a capillary. In order to maintain the spherical shape and the constant density of the particles they were kept in the glycerol water mixtures after their formation. In order to distinguish the particles visually according to their size, they were pigmented with four colors, i.e. a certain color for each diameter. The particle