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Interlaminar stresses of laminated composite beams resting on elastic foundation subjected to transverse loading

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ABSTRACT

In the present study, static analysis of the interlaminar stresses and free edge effects in a laminated composite beam resting on the Winkler-type elastic foundation is discussed. Equilibrium equations under transverse load along with the appropriate boundary conditions are obtained by using the Reddy's layerwise theory. An approximate elasticity solution for a special case of the boundary conditions is developed to verify the validity and accuracy of the present theory. Various examples are presented for the interlaminar normal and shear stresses along the interface and through the thickness of the beam and numerical results for the free edge-clamped boundary conditions are obtained. Finally, the effect of various parameters containing the concentration of the load, stiffness of the elastic foundation and dimension aspect ratio on the interlaminar stresses is studied. Results show an excellent agreement between the layerwise theory and approximate elasticity solution and demonstrate that the magnitude of interlaminar stresses in this problem are significant and should be considered in the structural design.

1. Introduction

One of the major problems in the application of the composite materials is the existence of delamination. This problem that arises due to the interlaminar stresses, commonly appears in the free edge of the structures and also in other special cases such as internal ply-drop, tapered laminates and geometric discontinuities. To determine the interlaminar stresses in the free edge of composite laminates, a three dimensional elasticity boundary value problem must be solved. But in general case, exact solution to this problem has not found and therefore most researches in this field include approximate methods.

Analysis of composite beams can be classified in two approaches. The first one neglects the deformation components in the width direction and results in simpler equations [1–4]. In the second approach, the normal stress component in the width direction is neglected however, it's effect is taken into account due to stress–strain and strain-deformation relations [4–8]. The second approach is more accurate and is used in the present study.

There are many numerical and analytical studies in the literature concerning to the calculation of the interlaminar stresses in the structures. The first analytical solution was proposed by Puppo and Evensen [9]. They assumed a finite-width composite laminate with isotropic shear layers and neglected the normal stress through the laminate. An approximated elasticity solution for the extension problem of the laminated composites was developed by Pipes and Pagano [10]. A perturbation method was used by Hsu and Herakovich [11] for the extension problem of the angle-ply composite laminates. Wang and Choi [12] used the Likhnitskii's stress potential and the theory of anisotropic elasticity to discuss about the singularity of stresses in the free edge of laminate. Wu and Kuo [13] analyzed a composite laminate under cylindrical bending with a local higher-order lamination theory. Kim and Atluri [14] used an approximate method based on the equilibrated stress representations and the principle of minimum complementary energy for analyzing interlaminar stresses in a beam-type composite laminate under out-of-plane shear/bending. Tahani and Nosier [15] determined interlaminar stresses resulting from bending of rectangular cross-ply composite laminates by using a layerwise laminate theory. By using this theory, Nosier and Bahrami [16] analyzed the free edge stresses in antisymmetric angle-ply laminates in extension and torsion and Nosier and Maleki [17] studied the free edge stresses of composite laminates with general stacking sequences. Interlaminar stresses and free edge effects in a sandwich panel with flexible core and laminated composite face sheets under transverse loads were studied by Afshin et al. [18,19]. Based on a three-dimensional multi-term extended Kantorovich method, interlaminar stresses of

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the general rectangular thick laminated plates was analyzed by Yazdani Sarvestani [20]. A multicontinuum approach was used by Nelson et al. [21] to analyze composite laminates subjected to hydrostatic stresses. Based on the reduced from of elasticity displacement field for a long laminate, an analytical method was established by Andakhshideh and Tahani [22] to exactly obtain the interlaminar stresses near the free edges of generally laminated composite plates. Huang and Kim [23] studied free edge interlaminar stresses of piezo-bonded composite laminates under symmetric electric excitation field.

Most of the analytical solutions can only be used for the simple geometries. Therefore, researchers have tried to evaluate interlaminar stresses by using the numerical methods such as finite-element and finite-difference. A three dimensional finite-difference solution developed by Altus et al. [24] to study the edge-effect in the angle-ply laminates. Spilker and Chou [25] studied interlaminar stresses by using hybrid-stress finite-element method. Murthy and Chamis [26] used a three dimensional finite-element method for analyzing the interlaminar stresses in composite laminates under various loading.

Composite beams or plates resting on elastic foundation were studied by using two approaches includes one parameter or Winkler-type and two parametric or Pasternak elastic foundation. The first one takes into account the transverse deflection of the beam or plate and the second one takes into account the transverse deflection as well as shear between laminate and elastic foundation. Many problems are studied by using these methods that some of recent works are referred here. Large amplitudes free vibrations and post-buckling analysis of unsymmetrically laminated composite beams on linear and nonlinear elastic foundation was studied by Baghani et al. [27]. They employed variational iteration method to solve the governing equations. Yu et al. [28] analyzed the propagation of flexural wave in periodic beam on elastic foundations by the transfer matrix method. Using a refined hyperbolic shear deformation theory, delamination of layered structures on elastic foundation was studied by Nedri et al. [29].

The results of researches demonstrate that the interlaminar stresses in the extension problem are significant. But these stresses in the transverse loads are not considerable. One of the special cases that these stresses are significant and has not investigated until now is the edge effect problem of laminated composite beams resting on an elastic foundation. In this work, at first, interlaminar stresses in a general laminated beam resting on Winkler-type elastic foundation is studied by using the layerwise theory of laminates. Then to verify the results of layerwise theory, an approximate elasticity solution for a special case of the boundary conditions is developed and it's results is compared with those of the layerwise theory. At the end, a parametric study for different ply stacking in the case of free edge-clamped boundary conditions is conducted and the effect of variation of parameters on the interlaminar stresses by using the layerwise theory is studied.

2. Layerwise formulation

Consider a laminated beam resting on an elastic foundation that subjected to a transverse load $P_z(x)$. The geometry of the beam is shown in Fig. 1. In this study a Winkler-type foundation is assumed. In a Winkler-type foundation, the pressure developed at any point between the beam and the foundation is proportional to the deflection of the beam at that point.

2.1. Displacement field and strains

The displacement field that is assumed in this study is independent of coordinate in width direction of the beam. This displacement field according to the layerwise theory may be illustrated as [30]:

$$\begin{aligned} u_1(x,y,z) &= U_k(x)\Phi_k(z) \\ u_2(x,y,z) &= V_k(x)\Phi_k(z) \qquad k=1,2,\ldots,N+1 \\ u_3(x,y,z) &= W_k(x)\Phi_k(z) \end{aligned} \tag{1}$$

where N is the number of numerical layers that is considered through the thickness of laminate and there is a summation on repeated index k. In Eq. (1), u_1, u_2 and u_3 are the displacement components of material point in the laminate in the x, y and z directions, respectively. Also U_k, V_k and W_k are the displacement components of all points located on the kth numerical plane of laminate in the x, y and z directions, respectively. $\Phi_k(z)$ is the global interpolation function. This function is continuous through the thickness of laminate and defined as [30]:

$$\Phi_k(z) = \begin{cases} 0 & z \leqslant z_{k-1} \\ \Psi^2_{k-1} & z_{k-1} \leqslant z \leqslant z_k \\ \Psi^1_k & z_k \leqslant z \leqslant z_{k+1} \\ 0 & z \geqslant z_{k+1} \end{cases} \qquad k=1,2,\ldots,N+1 \tag{2}$$

In Eq. (2), Ψ^j_k ($j = 1, 2$) is the local lagrangian interpolation function associated with the jth node of the kth layer. This function can be linear or higher order function of thickness coordinate z. In this study a linear lagrangian interpolation function will be used that defined as:

$$\Psi^1_k = \frac{z_{k+1}-z}{h_k}, \qquad \Psi^2_k = \frac{z-z_k}{h_k} \tag{3}$$

where h_k and z_k are thickness and z-coordinate of the bottom of the kth numerical layer, respectively. The displacement field of Eq. (1) has a C^0 continuity through the thickness of laminate. Therefore the transverse strain components are not continuous at the interface of two layers. This allows the interlaminar stresses to be continuous at the interface of two layers. From the substitution of Eq. (1) in the linear strain–displacement relation of elasticity, strain components will be obtained as:

$$\begin{aligned} &\varepsilon_x = \tfrac{dU_k}{dx}\Phi_k, \quad \varepsilon_y = 0, \quad \varepsilon_z = W_k\tfrac{d\Phi_k}{dz} \\ &\gamma_{yz} = V_k\tfrac{d\Phi_k}{dz}, \quad \gamma_{xz} = U_k\tfrac{d\Phi_k}{dz} + \tfrac{dW_k}{dx}\Phi_k, \quad \gamma_{xy} = \tfrac{dV_k}{dx}\Phi_k \end{aligned} \tag{4}$$

2.2. Equations of equilibrium

Principle of virtual work for a beam resting on elastic foundation may be written as:

$$\delta U_b + \delta U_f + \delta V = 0 \tag{5}$$

where δU_b and δU_f are the internal virtual works in the beam and foundation, respectively and can be written as:

$$\begin{aligned} \delta U_b = \int_{-b/2}^{b/2}\int_0^L\int_{-h/2}^{h/2} &(\sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \sigma_z\delta\varepsilon_z + \sigma_{yz}\delta\gamma_{yz} \\ &+ \sigma_{xz}\delta\gamma_{xz} + \sigma_{xy}\delta\gamma_{xy})dzdxdy \end{aligned} \tag{6}$$

and

$$\delta U_f = \int_{-b/2}^{b/2}\int_0^L (k_f W_{N+1}\delta W_{N+1})dxdy = bk_f\int_0^L \Delta^{kj} W_j \delta W_k dx \tag{7}$$

where k_f is the stiffness of the foundation (i.e. force/length^3) and Δ^{kj} is defined as:

$$\Delta^{kj} = \delta_{k(N+1)}\delta_{j(N+1)} \qquad k,j = 1,2,\ldots,N+1 \tag{8}$$

In Eq. (5), δV is the external virtual work due to transverse load $P_z(x)$ applied on the top surface of laminate and can be written as:

$$\delta V = -\int_{-b/2}^{b/2}\int_0^L P_z(x)\delta W_1(x)dxdy \tag{9}$$