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Flow of wet powder in a conical centrifugal filter—an analytical model

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A R T I C L E I N F O

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A B S T R A C T

A one-dimensional analytical model is developed for the steady state, axisymmetric, slender flow of saturated powder in a rotating perforated cone. Both the powder and the fluid spin with the cone with negligible slip in the hoop direction. They migrate up the wall of the cone along a generator under centrifugal force, which also forces the fluid out of the cone through the powder layer and the porous wall. The flow thus evolves from an over-saturated paste at inlet into a nearly dry powder at outlet. The powder is treated as a Mohr–Coulomb granular solid of constant void fraction and permeability. The shear traction at the wall is assumed to be velocity and pressure dependent. The fluid is treated as Newtonian viscous. The model provides the position of the *colour line* (the transition from over- to under-saturation) and the flow velocity and thickness profiles over the cone. Surface tension effects are assumed negligible compared to the centrifugal acceleration. Two alternative conditions are considered for the flow structure at inlet: fully settled powder at inlet, and progressive settling of an initially homogeneous slurry. The position of the colour line is found to be similar for these two cases over a wide range of operating conditions. Dominant dimensionless groups are identified which control the position of the colour line in a continuous conical centrifuge. Experimental observations of centrifuges used in the sugar industry provide preliminary validation of the model.

1. Introduction

Centrifugal filters are commonly used in the food processing and chemical industries in order to separate the liquid and solid phases of a mixture, where the two phases have comparable densities. There exist two main types of centrifugal filter: *batch* machines with a cylindrical basket and *continuous* machines with a conical basket. The present study focuses on continuous conical centrifuges, which are most commonly used in the sugar industry to separate sucrose crystals from molasses. Swindells (1982) and Greig (1995) extensively studied these machines in a semi-empirical fashion. While their work provides valuable insight in the operation of conical centrifuges in the sugar industry it does not fully address the underlying mechanics. Consequently, only a limited number of operating parameters can be used to optimize the sugar machine and application of their results to pharmaceutical, chemical or other food products is difficult. This study aims at developing a more general model of the continuous centrifugal filter.

The operation of a continuous centrifuge in the sugar industry is now described. The rotating basket of the machine, sketched in Fig. 1, is conical with a jump in cone angle along its length: a lower impervious cone has a semi-angle of $\alpha=15^{\circ}$ whereas the upper perforated cone has $\alpha=30^{\circ}$. The basket is about 1 m in diameter at outlet and spins at 1000 RPM to provide a maximum centripetal acceleration of 500g. The inside wall of the upper, perforated cone is fitted with a slotted screen, thereby allowing for fluid drainage but preventing crystal losses. Slip of the sugar crystals against the screen is favoured by the use of smooth perforated screens with narrow slots and open areas of only 10–15%. The feedstock, in the form of a sugar/molasses slurry (*massecuite*), of liquid volume fraction 50% and temperature 60 °C, is introduced along the spin axis into the lower impervious cone at a constant flow rate. The slurry acquires the angular velocity of the basket and migrates up the wall of the lower cone into the upper, perforated cone under centrifugal force. In an initial region, labelled region I in Fig. 2, the flow is still over-saturated. Liquid drainage causes the slurry to quickly evolve into a cake of densely packed powder, the top of which is damp while the bottom is still saturated with liquid (region II). Finally, in region III only a residual liquid fraction remains and the flow consists of a cake of damp powder sliding over the screen. Region II is the transition zone between flow of over-saturated powder (region I) and flow of damp powder (region III) and is commonly called the *colour line* in the sugar industry. Further liquid drainage in region III is assumed to be negligible and the damp powder is therefore treated as a homogeneous continuous medium of constant properties.

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While the microstructure of the flow in regions II and III is clear, in region I it depends largely on the operating conditions as we shall now discuss. In the current study we consider the case where the particles are denser than the fluid, so that they tend to settle onto and slide against the wall instead of floating on top of the fluid. At the start of the upper, perforated cone a fraction of the particles may have already settled onto the wall and the remaining powder may not be homogeneously distributed in the overlying fluid. In this analysis we shall consider two idealized extremes: in a first case all the solid particles are assumed settled and sliding before the flow reaches the inlet of the perforated cone; in a second case the flow is still a homogeneous slurry at inlet, with only minimal initial sedimentation. Although the actual flow condition at inlet is expected to be somewhere between these two extremes, we will show that the difference in position of the colour line resulting from these two cases is small. The microstructure considered in each of the three regions is shown in Fig. 3.

In order to develop an analytical model for the flow in a continuous conical centrifuge we will first present some results from the literature. Based on these results we then develop a one-dimensional model of the flow in a continuous conical centrifuge.

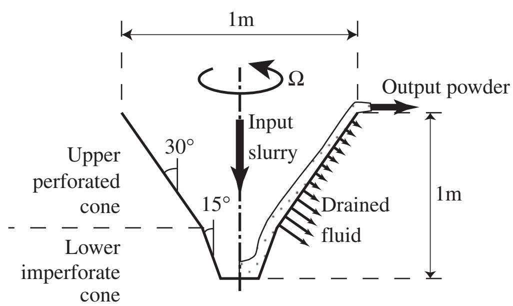

Fig. 1. Section of a typical sugar conical continuous centrifuge.

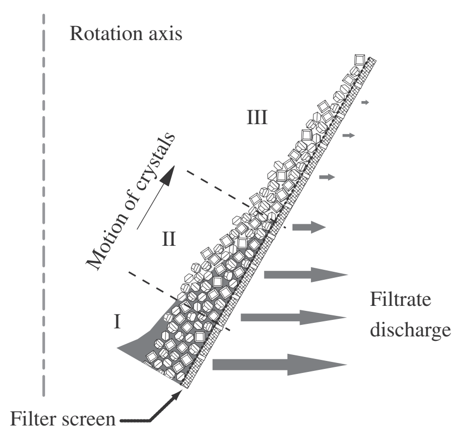

Fig. 2. Flow in a rotating perforated cone.

This model is then non-dimensionalized and solved numerically for a typical application of the conical centrifuge: the separation of liquid molasses from sugar crystals in the sugar industry. Preliminary validation of the model is provided by observation of an industrial centrifuge.

2. Model assumptions

2.1. Slender centrifugal flow

A spherical co-ordinate system (r,θ,ϕ) (see Fig. 4) is appropriate for describing the flow in a conical centrifugal filter. In most practical situations the flow in the cone is slender: its thickness h is significantly smaller than the radial co-ordinate r. A boundary layer approximation of the momentum equations is therefore applicable. This approach to analysis of flow in a spinning cone has been adopted by Bruin (1969) and Makarytchev et al. (1997, 1998) for a Newtonian viscous fluid, and by Symons (2011b) for a damp powder.

The relative importance of convective, Coriolis and centripetal accelerations in the momentum equations (when written in a rotating frame of reference) may be assessed via the Rossby number Ro as defined by (Makarytchev et al., 1997)

$$Ro = \frac{u}{r\Omega \sin\alpha} \approx \frac{\text{convective}}{\text{Coriolis}} \approx \frac{\text{Coriolis}}{\text{centripetal}} \tag{1}$$

where u is the through-thickness averaged radial velocity, Ω the angular velocity of the cone and α is the cone apex angle. In this study we assume that the Rossby number is significantly smaller than unity, so that both convective and Coriolis accelerations are negligible compared to the centripetal acceleration. This assumption is in accordance with observations of typical slender, high viscosity flows observed in industrial conical centrifuges. A consequence of a small Rossby number (i.e. low radial velocity) is that the circumferential slip (in the ϕ-direction) is negligible and therefore the flow has virtually the same angular velocity as the cone at any radius r (see e.g. Symons, in press, 2011b).

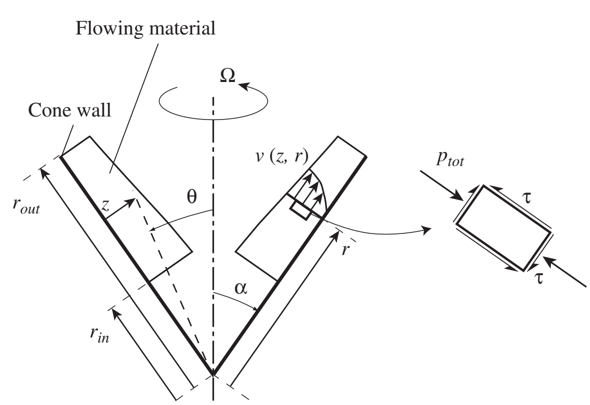

Fig. 4. Model geometry.

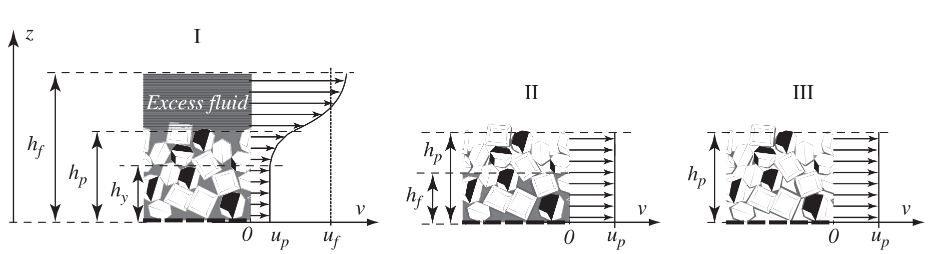

Fig. 3. Microstructure and velocity profile of the solid/liquid/air phases in regions I, II and III. The excess fluid can be pure interstitial fluid (case 1) or a homogeneous mixture of interstitial fluid and powder (case 2).