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A multi-scale computational method including contact for the analysis of damage in composite materials

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ABSTRACT

In this article, a multi-scale computational homogenization scheme is proposed for the study of composite materials. A classical unilateral contact law has been incorporated in the microscopic level, for the investigation of the contact between the constitutive materials. The either-or decision resulting from the contact-no contact condition in the microscopic scale, makes the problem non-linear. This change in the contact state of the microscopic level, is taken into account by the proposed approach. Debonding between the matrix and the surrounding fibers and its impact on the macroscopic structure, are depicted. In addition, a change in the direction of the macroscopic load during analysis, results in a non-linear behavior due to the alteration of the microscopic contact state. The distribution of the displacement jump is influenced in this case, as well.

1. Introduction

In the present work a multi-scale, computational homogenization scheme is proposed for the study of composite materials. A classical unilateral contact law is applied in the microscopic interfaces to capture the contact behavior of the constitutive materials.

Several different, analytical and numerical approaches have been proposed in the past for the investigation of complex, non-linear, heterogeneous structures, like composites. Analytical/mathematical methods, like asymptotic homogenization [1], can be more accurate in the description of the micro structure, for relatively simple microscopic patterns and constitutive laws. On the other hand, numerical methods may be used for the simulation of complex microscopic geometries, over a statistically defined representative amount of material [2].

Numerical/computational homogenization can be extended to cover several non-linear effects, like contact, debonding, damage and plasticity [3]. According to numerical homogenization, a unit cell is explicitly solved and the resulting average quantities are then used for the determination of the parameters of a macroscopic constitutive law [4,5].

From another point of view, multi-scale computational homogenization incorporates a concurrent analysis of both the macro and the microstructure in a nested multi-scale approach, [6–14]. Within this method, the macroscopic constitutive behavior is determined during simulation, after solving the microscopic problem and transferring the necessary information on the macroscopic scale. This approach, which is generally called FE^2, offers the flexibility of simulating complex microstructural patterns, with every kind of non-linearity. Furthermore, the evolution of the microscopic structure can be taken into account, by using this method. More recently, some sophisticated efforts for investigating localization phenomena with computational homogenization tools, appear in the literature [15–20]. It is worth noticing that in the majority of these articles, a continuous damage law has been used to simulate failure in the microscopic scale. In the macroscopic scale a discontinuous law describing a macro crack is numerically obtained.

Some efforts toward coupling contact mechanics and computational homogenization also appear. In [21,22] contact mechanics is coupled with multi-scale homogenization for the study of rough surfaces while in [23] the investigation of a three body frictional system with rigid particles, embedded between a deformable elastic solid and a rigid surface, takes place. Debonding between the matrix and the surrounding inclusions of a composite material has been also studied in [24] by using contact mechanics and homogenization concepts. In [25,26] an interfacial failure model with normal and tangential brittle-elastic springs and a bi-linear

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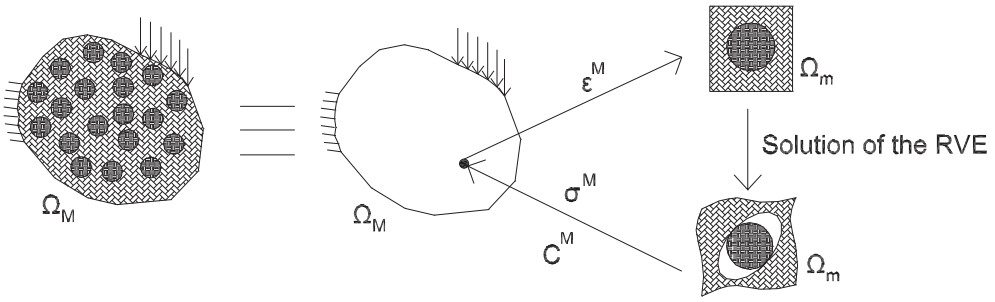

Fig. 1. Schematic representation of the multi-scale, computational homogenization.

cohesive zone model have been developed respectively, for the simulation of progressive debonding in the matrix-fiber interface of composite materials. In [27] a bilinear cohesive law is used to simulate failure between the fibers and the matrix interface; then, a comparison between periodic and minimal kinematic boundary conditions is given. In [28] a unilateral contact-friction and a damage evolution law have been used for the simulation of a masonry, brick–mortar interface, within a homogenization framework.

In this article an Augmented Lagrangian formulation has been used for the simulation of the contact state between the constitutive materials, in the microscopic scale of composite structures. A continuous macroscopic model is considered in the macro scale and an overall multi-scale contact computational homogenization scheme is developed. With this numerical scheme, several phenomena related to the microscopic contact conditions and its impact on the macroscopic model, have been investigated. Among them are included the influence of the jump of displacements on the macroscopic response, as well as the alteration of the macroscopic load direction during analysis resulting in a microscopic contact change and its impact on the macroscopic structural behavior.

2. A short introduction to computational homogenization

The approach adopted in this article is related to the concurrent analysis of the macroscopic and the microscopic structure, respectively. According to the classical formulation of the multi-scale computational homogenization [6–8,10], two nested boundary value problems are concurrently solved. The initial heterogeneous macroscopic structure is equivalent with a homogeneous one, in each Gauss point of which, a suitably defined RVE is correlated. This RVE includes every heterogeneity and non-linearity of the material.

With linear or periodic boundary conditions, a macroscopic strain is the loading of the RVE. After analysis and convergence of each RVE in every Gauss point, results concerning the average stress and the stiffness are given back to the macroscopic structure, Fig. 1. No assumption for the constitutive law of the macroscopic structure is a priori considered, thus the macroscopic constitutive behavior is numerically obtained. This is a practical solution to the major question of homogenization, namely which are the properties of the homogeneous constitutive law.

3. Averaging relations

According to the Hill–Mandel condition or energy averaging theorem, the macroscopic volume average of the variation of work equals to the local work variation, on the RVE [29]:

$$\boldsymbol{\sigma}^M : \delta\boldsymbol{\epsilon}^M = \frac{1}{V_m}\int_{V_m} \boldsymbol{\sigma}^m : \delta\boldsymbol{\epsilon}^m dV_m \tag{1}$$

Among others, three widely used types of loading states, which satisfy the above condition, can be applied to the RVE: (a) prescribed linear displacements, (b) prescribed tractions, (c) periodic boundary conditions. In the present study both linear displacement and periodic boundary conditions have been used.

According to linear displacement boundary conditions, the loading in the boundaries of the RVE is given by the following relation:

$$\mathbf{u}|_{\partial V_m} = \boldsymbol{\epsilon}^M \mathbf{x} \tag{2}$$

where a loading strain $\boldsymbol{\epsilon}^M$ is applied to the boundaries ∂V_m of the RVE. With $\mathbf{x}$ is denoted the matrix with the undeformed coordinates of the boundary nodes of the RVE.

Periodic boundary conditions require periodic displacements, as well as antiperiodic tractions, in the opposite boundaries of the RVE. In particular, the displacements of the opposite boundaries are given by the following equations:

$$\mathbf{u}_T - \mathbf{u}_B = \mathbf{u}_4 - \mathbf{u}_1 \tag{3a}$$

$$\mathbf{u}_L - \mathbf{u}_R = \mathbf{u}_1 - \mathbf{u}_2 \tag{3b}$$

where the displacements in the top, bottom, left and right boundary are estimated by using the prescribed displacements of three corner nodes of the RVE, namely 1, 2 and 4, given by relation (2).

In order to proceed in the formulation of a homogenization scheme, the average quantities of both the microscopic strain and stress should be defined. The constitutive relation will be then numerically built, as is shown later in this article. The general averaging relations, are:

$$\langle\boldsymbol{\epsilon}\rangle_{V_m} = \frac{1}{V_m}\int_{V_m} \boldsymbol{\epsilon}^m dV_m, \quad \langle\boldsymbol{\sigma}\rangle_{V_m} = \frac{1}{V_m}\int_{V_m} \boldsymbol{\sigma}^m dV_m \tag{4}$$

Eq. (4) can be further simplified. The volume average microscopic strain is equal to the macroscopic strain which has been applied as loading to the boundaries of the RVE:

$$\langle\boldsymbol{\epsilon}\rangle_{V_m} = \boldsymbol{\epsilon}^M \tag{5}$$

In case prescribed displacements are applied to the RVE, the following simplified formulation for the macroscopic stresses, has been chosen [10,13]:

$$\langle\boldsymbol{\sigma}\rangle_{V_m} = \frac{1}{V_m}\mathbf{f}\mathbf{x} = \boldsymbol{\sigma}^M \tag{6}$$

where $\mathbf{f}$ is the matrix of the resulting external forces in the undeformed coordinates of the boundary nodes $\mathbf{x}$ of the RVE, after microscopic analysis has been completed.

A similar relation is chosen for the macroscopic stress, in case periodic boundary conditions are used [10,13]:

$$\langle\boldsymbol{\sigma}\rangle_{V_m} = \frac{1}{V_m}\mathbf{f}_p\mathbf{x}_p = \boldsymbol{\sigma}^M \tag{7}$$