Computational Materials Science 94 (2014) 150–162

Contents lists available at ScienceDirect

Computational Materials Science

journal homepage: www.elsevier.com/locate/commatsci

Determination of the effective conductive properties of composites with curved oscillating interfaces by a two-scale homogenization procedure

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A R T I C L E I N F O

Article history:
Received 25 December 2013
Received in revised form 6 March 2014
Accepted 9 March 2014
Available online 13 April 2014

Keywords:
Micromechanics
Composites
Thermal conductivity
Asymptotic analysis
Curved rough interface

A B S T R A C T

Rough surfaces and interfaces appear in many situations of practical interest in physics and mechanics of solids. When the interfaces between the constituent phases of composites are rough instead of being smooth as assumed usually, all well-known micromechanical schemes resorting to Eshelby's tensor are no longer applicable to computing their effective properties. The present work proposes a two-scale homogenization procedure aiming at determining the effective thermal properties of a two-dimensional composite in which the curved interfaces between the constituent phases oscillate periodically and quickly. An asymptotic analysis method is first used to homogenize a rough interface zone as an equivalent interphase, and the effective thermal conductivity tensor of this interphase at the mesoscopic scale is exactly determined. Then, by applying two micromechanical schemes, closed-form expressions for the effective thermal properties of composites at the macroscopic scale are derived. Finally, the analytical results obtained are compared with the relevant bounds and with the corresponding numerical results provided by the finite element method. The two-scale homogenization procedure turns out to be accurate and efficient.

1. Introduction

The studies dedicated to estimating the effective properties of inhomogeneous materials in terms of local phase properties and their microstructure often adopt two assumptions about the interfaces between the constituent phases: (i) they are smooth and (ii) they are perfect bondings. When these two assumptions hold, micromechanical schemes resorting to the classical Eshelby's results about an elliptic or ellipsoidal inclusion embedded in an infinite matrix, such as dilute, Mori–Tanaka, self-consistent and generalized self-consistent schemes, can be applied to determine the effective properties of inhomogeneous materials. For more details about these micromechanical schemes, the reader can refer to the review paper of Hashin [1]. However, many situations in practice occur where the assumption of perfect bonding or the assumption of smooth interfaces is inappropriate. Relaxation of the assumption of interfacial perfect bonding has been made, giving rise to a rich literature on imperfect smooth interfaces (see, e.g., [2,3] and the relevant references cited therein). In the context of thermal conduction, the most widely used imperfect interface models are Kapitza's thermal resistance model and the highly conducting interface one (see, e.g., [4,5]).

Relaxation of the second assumption of smooth interfaces has also been made, leading to a number of studies in the fields of physics and mechanics of solids or fluids devoted to rough surfaces and rough perfect interfaces (see, e.g., [6–16]). In general, a surface or an interface which is smooth at a given scale becomes rough at a smaller scale. In the case where rough interfaces are involved, most of the micromechanical schemes proposed in the literature for determining the effective properties of inhomogeneous materials fail to be valid, at least from an analytical point of view. Indeed, as mentioned above, most of them need using Eshelby's results about an elliptic or ellipsoidal inclusion embedded in an infinite matrix. These results are no long valid when the interface between an inclusion and the matrix is rough. This observation constitutes the main motivation of the present work. Note that the numerical micromechanics or homogenization methods in which Eshelby's

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http://dx.doi.org/10.1016/j.commatsci.2014.03.014
0927-0256/

results are not invoked are still valid even though rough interfaces intervene. However, when interfaces are very rough, the numerical methods applied to treating them may be very time-consuming.

According to the roughness amplitude of an interface or a surface, the relevant problem can be analyzed either by using certain perturbation techniques when the roughness amplitude is much smaller than its wavelength (see, e.g., [17] and the relevant references cited therein) or studied by applying some homogenization approaches when the wavelength is much smaller than the amplitude (see, e.g., [18–32]).

Remark that relaxation of both the interfacial smoothness and perfect bonding assumptions seems not have been made in the literature.

The present work is concerned with the effective conductivity of two-dimensional composites consisting of a matrix in which inclusions are embedded via perfect interfaces oscillating periodically and quickly about a curve. In this work, a two-scale homogenization procedure is proposed to estimate the effective thermal conductivity of these composites. First, at the mesoscopic level, a mathematical asymptotic analysis is performed to homogenize as an equivalent interphase an interface zone in which the rough interface oscillates. Remarkably, the effective conductivity of the equivalent interphase can be determined exactly. Second, at the macroscopic level, closed-form expressions are obtained for the effective conductivity of the composite by using the two well-known micromechanical models, namely the coated ellipse assemblage (CEA) and the coated circle assemblage (CCA). These models are used to study two rough interface configurations in detail. The first configuration is a rough interface exhibiting a comb profile and oscillating along an elliptical curve. In the second configuration, the interface possesses a saw-tooth profile and oscillating along a circular curve. The corresponding analytical results obtained by the two-scale homogenization method are finally compared with the numerical results provided by the finite element method as well as with the Reuss, Voigt and Hashin–Shtrikman bounds. These comparisons confirm the validity of the proposed two-scale homogenization method.

The paper is organized as follows. Section 2 is dedicated to specifying the setting of the problem under investigation. In Section 3, by carrying out an asymptotic analysis, a rough interface zone is homogenized and replaced by an equivalent interphase whose effective conductivity is analytically and explicitly obtained. In Section 4, the closed-form expressions for the macroscopic conductive properties of the composite under consideration are determined by using CEA and CCA. In Section 5, the derived analytical results are compared with the relevant bounds and with the corresponding numerical results obtained by the finite element method. Finally, a few concluding remarks are given in Section 6.

2. Problem setting

We consider a domain Ω formed of two sub-domains $\Omega^{(1)}$ and $\Omega^{(2)}$ perfectly bonded together at their interface Γ in the two-dimensional context. Letting (x_1, x_2) be a system of Cartesian coordinates associated to an orthonormal basis $\{\mathbf{j}_1, \mathbf{j}_2\}$, we are interested in the case where Γ corresponds to a periodically oscillating curve in the x_1–x_2 plane (see Fig. 1). To obtain a mathematical characterization of Γ, the x_1–x_2 plane is parameterized by two orthogonal curvilinear coordinates y_1 and y_2 such that the position vector $\mathbf{x}$ of any point in the x_1–x_2 plane is given by

$$\mathbf{x} = \mathbf{x}(y_1, y_2) = [x_1(y_1, y_2), x_2(y_1, y_2)].$$

The vector tangent to the y_i-coordinate curve is defined by

$$\mathbf{t}_i = \frac{\partial \mathbf{x}}{\partial y_i} = h_i \mathbf{f}_i \quad \text{with} \quad h_i = \left\| \frac{\partial \mathbf{x}}{\partial y_i} \right\|$$

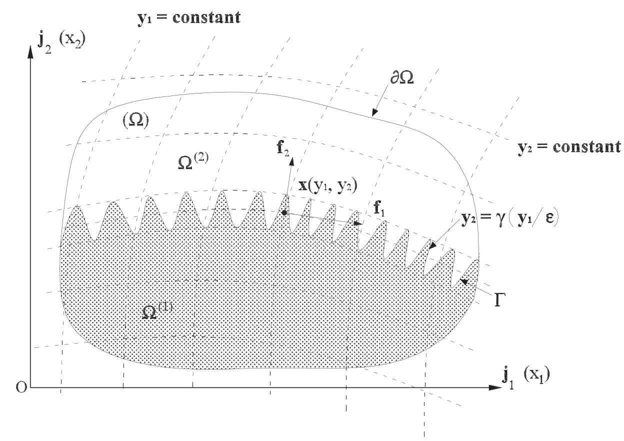

Fig. 1. The two-dimensional domain Ω consists of two phases occupying the subdomains $\Omega^{(1)}$ and $\Omega^{(2)}$ with their rough interface Γ oscillating periodically along a curve characterized by $y_2 = \gamma(y_1/\epsilon)$.

where the summation convention does not apply, h_i is a metric coefficient and $\mathbf{f}_i$ is the unit vector tangent to the y_i-coordinate curve. Since the curvilinear coordinates y_1 and y_2 are orthogonal, $\mathbf{f}_1$ is perpendicular to $\mathbf{f}_2$ so that $\mathbf{f}_1 \cdot \mathbf{f}_2 = 0$. The interface Γ between $\Omega^{(1)}$ and $\Omega^{(2)}$ is now defined by

$$\Gamma = \left\{ \mathbf{x} = \mathbf{x}(y_1, y_2) \in \mathbf{R}^2 \mid y_2 = \gamma(\alpha), \quad \alpha = \frac{y_1}{\epsilon} \right\}. \tag{1}$$

In this equation, ϵ is a small positive parameter and $\gamma(\alpha)$ with $\alpha = y_1/\epsilon$ is a periodic function of period 1. Denoting the minimum and maximum values of $\gamma(\alpha)$ by γ_{min} and γ_{max}, respectively, we assume that $0 < \epsilon \ll \delta = \gamma_{max} - \gamma_{min}$. This means that Γ is a very rough interface oscillating periodically about the y_1-coordinate curve. In addition, it is assumed that, for any given value $\bar{y}_2$ of y_2 such that $\gamma_{min} < \bar{y}_2 < \gamma_{max}$, equation $\bar{y}_2 = \gamma(\alpha)$ has two distinct real roots α_1 and α_2 within a period, namely $\bar{y}_2 = \gamma(\alpha_1) = \gamma(\alpha_2)$ with $0 \leqslant \alpha_1 < \alpha_2 < 1$.

For later use, we denote by $\omega^{(c)}$ the rough interface zone characterized by

$$\omega^{(c)} = \{ \mathbf{x} = \mathbf{x}(y_1, y_2) \in \Omega \mid \gamma_{min} < y_2 < \gamma_{max} \}, \tag{2}$$

and by $\pi(\bar{y}_2)$ the curved line defined by

$$\pi(\bar{y}_2) = \left\{ \mathbf{x} = \mathbf{x}(y_1, y_2) \in \omega^{(c)} \mid y_2 = \bar{y}_2, \gamma_{min} < \bar{y}_2 < \gamma_{max} \right\}. \tag{3}$$

Relative to the curvilinear coordinate system $\{y_1, y_2\}$ associated with the orthonormal curvilinear basis $\{\mathbf{f}_1, \mathbf{f}_2\}$, the thermal conduction behavior of the material forming the sub-domaine $\Omega^{(p)}$, with $p = 1$ or 2, is described by Fourier's law

$$\mathbf{q}^{(p)} = -\mathbf{K}^{(p)} \cdot \nabla \theta^{(p)}. \tag{4}$$

Here $\mathbf{q}^{(p)}$ and $\theta^{(p)}$ stand for the heat flux and temperature fields, respectively, and $\mathbf{K}^{(p)}$ is the second-order thermal conductivity tensor of phase p. By hypothesis, the both materials are curvilinearly anisotropic, but may be heterogeneous along y_2-direction and periodically heterogeneous along the y_1-direction with the same period ϵ as the interface Γ, so that $\mathbf{K}^{(p)}(y_1, y_2) = \mathbf{K}^{(p)}(y_1 + \epsilon, y_2)$. In (4), the temperature gradient $\nabla \theta^{(p)}$ is calculated in the orthonormal curvilinear basis $\{\mathbf{f}_1, \mathbf{f}_2\}$ by

$$\nabla \theta^{(p)} = \theta^{(p)}_{,1} \mathbf{f}_1 + \theta^{(p)}_{,2} \mathbf{f}_2 \tag{5}$$

where the derivatives $(\bullet)_{,1}$ and $(\bullet)_{,2}$ are, hereafter, defined as

$$(\bullet)_{,1} = \frac{1}{h_1} \frac{\partial (\bullet)}{\partial y_1}, \quad (\bullet)_{,2} = \frac{1}{h_2} \frac{\partial (\bullet)}{\partial y_2}. \tag{6}$$

The local heat flux $\mathbf{q}^{(p)}$ satisfies the energy conservation equation