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Piezoelectric composites: Imperfect interface models, weak formulations and benchmark problems

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ABSTRACT

This work is concerned with piezoelectric composites with imperfect interfaces. The general piezoelectric imperfect interface model derived from the replacement of a piezoelectric interphase of small uniform thickness between two bulk piezoelectric phases by an imperfect interface through an asymptotic analysis is reformulated so as to obtain simpler compact characteristic expressions. Further, it is exploited to deduce two particular piezoelectric imperfect interface models which include as special cases the widely used Kapitza model, highly conducting model, elastic spring-layer model and membrane-type (or Gurtin–Murdoch) model. The weak formulations for the mixed boundary value problems of a piezoelectric composite with the interfaces described by the derived particular piezoelectric imperfect interface models are provided and serve as the basis for their numerical treatment by the extended finite element method (XFEM). The analytical solutions for benchmark problems are found and can be used for carrying out the accuracy and convergence analyzes of numerical methods such as XFEM.

1. Introduction

In a great number of practical situations, the interfaces between the dissimilar phases in composites are imperfect and may greatly affect their local field distributions and overall properties [see, e.g., 1–4]. In the context of uncoupled phenomena such as thermal conduction and elasticity, three models have been mainly proposed and developed for imperfect interfaces. In the setting of thermal conduction, the first model is Kapitza's thermal resistance one which is widely adopted [5,6]. According to this model, the temperature suffers a jump across an interface but the normal heat flux is continuous and proportional to the temperature jump across the same interface. The second model is the highly conducting interface one which is adopted in studying the effective conductivity of some composites [see, e.g., 7–9]. This model stipulates that the temperature is continuous across an interface while the normal heat flux presents a interfacial jump which is related to the surface temperature gradient by the Laplace–Young equation. The third model, referred to as the general thermal imperfect interface model (see, e.g., [10,11]), is based on the idea of replacing an interphase by an imperfect interface. In this model, both the temperature and normal heat flux suffer a jump. In addition, it is shown that the first two imperfect interface models can be retrieved from the third general one in a mathematically rigorous manner by taking the interphase to be, respectively, weakly and highly conducting [10,11].

Within the framework of linear elasticity, the counterparts of the Kapitza interfacial thermal resistance model and the highly conducting interface model are the spring-layer and the membrane-type imperfect interface models which are widely used [see, e.g., 1,2,12]. In a similar way, these two elastic interface models can be derived from the elastic general interface model by considering soft and stiff interphases [11,13]. From the physical standpoint, the special interface models obtained for the cases where the properties of the interphase present a high contrast with respect to those of the surrounding phases are important. This is because, on the one hand, they are helpful for getting a good understanding of the general interface model and, on the other hand, they are directly useful for some situations of practical interest.

In the context of piezoelectricity, there are also three interface models: two special interface models and one general interface model. The first special interface model, proposed by [14], can be called as the piezoelectric membrane-type interface model which

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can be viewed as a piezoelectric extension the well-known Gurtin–Murdoch model [15]. According to this model, the displacement and electric fields are continuous across an interface, while the traction vector field and the normal electric displacement exhibit interfacial jumps which are proportional to certain surface derivatives of the displacement and electric fields. This model has been applied in studying the size-dependent effective piezoelectric moduli of nanocomposites [16]. The second special interface model, referred as to the piezoelectric spring-type one, stipulates that the traction vector and the normal electric displacement are continuous across an interface, while the displacement vectors and the electric field suffer interfacial jumps proportional to the traction vector and the normal electric displacement. The effect of piezoelectric spring-type interfaces in piezoelectric composites has been investigated in [17,4]. The general piezoelectric interface model was derived by [18,19] by exploiting the idea of replacing an interphase by an imperfect interface and by an asymptotic analysis. In this model, the displacement vector, the electric field, the traction vector and the normal electric displacement field are in general discontinuous across an interface. We observe that the two special piezoelectric interface models described before are proposed on a phenomenological basis while the general piezoelectric interface model is derived in a physically sound and mathematically rigorous way. However, the connection between the two special piezoelectric interface models and the general piezoelectric interface has not been clarified.

A boundary value problem with imperfect interfaces can be solved in general only numerically. If a finite element method is adopted, a weak formulation of the problem is a preliminary step toward its numerical treatment. Although a great number of works have been dedicated to numerically simulating imperfect interface phenomena in composites [see, e.g., 20–23], the interfacial models used are often limited to uncoupled phenomena such as elasticity and electric conduction. As discussed above, the piezoelectric imperfect interface models are much more complicated. To the best of the authors' knowledge, the numerical treatment of piezoelectric imperfect interfaces remains an open problem.

The objective of the present work is threefold. First, on the basis of the recent work of Gu and He [19], it aims to present a simple and compact coordinate-free formulation for the general piezoelectric imperfect interface model and to rigorously derive two special piezoelectric imperfect interface models. Second, it has the purpose of providing weak formulations for the boundary value problem of a piezoelectric composite in which the interfaces are described by either of the two special piezoelectric imperfect interface models. These weak formulations are indispensable to the numerical treatment of the problem, for example, by the extended finite element method (XFEM). Finally, a benchmark piezoelectric problem is analytically solved where a composite consisting of a circular cylindrical piezoelectric inhomogeneity embedded in an infinitely extended piezoelectric matrix via an imperfect interface is under a remote anti-plane mechanical loading and a remote in-plane electric loading. The analytical results can be used to test the accuracy and convergence rate of a numerical approach.

The paper is organized as follows. In Section 2, after specifying the field equations governing a piezoelectric composite, the general piezoelectric imperfect interface model is reformulated so as to obtain simple and compact characteristic expressions. From the general piezoelectric imperfect interface model, two special ones are derived. In Section 3, the weak formulations for a mixed boundary value problem of the piezoelectric composite are given while using the special piezoelectric imperfect interface models. Section 4 is dedicated to analytically solving a benchmark problem. Finally, in Section 5, a few concluding remarks are drawn.

2. Piezoelectric composites with imperfect interfaces

2.1. Governing fields equations

The composite under consideration consists of a matrix reinforced by inhomogeneities via imperfect interfaces. The matrix and inhomogeneities are assumed to be individually homogeneous and linearly piezoelectric. Precisely, their piezoelectric laws take the following form:

$$\begin{cases} \sigma_{ij} = C_{ijkl}\varepsilon_{kl} - \Pi_{kij}e_k, \\ d_i = \Pi_{ikl}\varepsilon_{kl} + K_{ij}e_j, \end{cases} \tag{1}$$

where σ_{ij} and ε_{ij} are the components of the stress and strain tensors; d_i and e_i are the components of the electric displacement and electric field vectors; C_{ijkl}, K_{ij} and Π_{kij} stand for the elastic, dielectric and piezoelectric moduli, respectively. These moduli have the following symmetry properties:

$$C_{ijkl} = C_{ijlk} = C_{klij}, \quad K_{ij} = K_{ji}, \quad \Pi_{kij} = \Pi_{kji}. \tag{2}$$

In addition, the strain tensor $\boldsymbol{\varepsilon}$ is related to the displacement vector $\boldsymbol{u}$ by

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left[\nabla\boldsymbol{u} + \nabla^T\boldsymbol{u}\right], \tag{3}$$

and the electric field vector $\boldsymbol{e}$ is derived from the scalar potential φ via

$$\boldsymbol{e} = -\nabla\varphi. \tag{4}$$

The equilibrium equations for the stress tensor and electric displacement vector in the absence of body forces and electrical sources state

$$div\ \boldsymbol{\sigma} = \boldsymbol{0}, \tag{5}$$
$$div\ \boldsymbol{d} = 0. \tag{6}$$

Referring, for example, to [24], uncoupled and coupled three-dimensional (3D) linear phenomena can be formulated in a unified way by introducing r divergence-free vector fields $\boldsymbol{J}_\alpha$ and r curl-free vector fields $\boldsymbol{E}_\alpha$ with $\alpha = 1, 2, \ldots, r(\geqslant 1)$. In the context of piezoelectricity, taking $r = 4$, the constitutive law (1) can be rewritten as the following compact form

$$\boldsymbol{J}_\alpha = \boldsymbol{L}_{\alpha\beta}\boldsymbol{E}_\beta \quad \text{or} \quad J_{\alpha i} = \boldsymbol{L}_{\alpha i\beta j}\boldsymbol{E}_{\beta j}, \tag{7}$$

where the Greek letters α and β are field subscripts assuming the values 1–4, the Latin letters i and j are space subscripts taking the values 1, 2 and 3, and the summation convention applies to every couple of repeated Greek or Latin subscripts. More precisely, the following definitions are adopted:

$$J_{1i} = \sigma_{1i}, \quad J_{2i} = \sigma_{2i}, \quad J_{3i} = \sigma_{3i}, \quad J_{4i} = d_i, \tag{8}$$

$$E_{\alpha i} = w_{\alpha,i}, \quad \boldsymbol{w} = (u_1, u_2, u_3, \varphi)^T, \tag{9}$$

$$[L_{\alpha i\beta j}] = \begin{bmatrix} C_{i1j1} & C_{i1j2} & C_{i1j3} & \Pi_{ji1} \\ C_{i2j1} & C_{i2j2} & C_{i2j3} & \Pi_{ji2} \\ C_{i3j1} & C_{i3j2} & C_{i3j3} & \Pi_{ji3} \\ \Pi_{i1j} & \Pi_{i2j} & \Pi_{i3j} & -K_{ij} \end{bmatrix}. \tag{10}$$

Concerning the interface between a generic inhomogeneity and the matrix, a generalized piezoelectric membrane-type interface or spring-type interface model will be adopted for its description. These two models can be deduced from a general piezoelectric interface model as will be shown in the next section.