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Numerical implementation of a new coupled cyclic plasticity and continum damage model

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ABSTRACT

If effective stress is replaced by true stress for calculation of damage, the problems which arise due to strain softening are eliminated. This consideration leads to the coupling between damage and plasticity models. Despite beneficial features of this consideration, different orders of complexity will appear in the numerical solution of the resulting constitutive equations. In this work, the fundamental equations for coupling damage with nonlinear cyclic plasticity model based on small deformation assumption are derived and the continuum relations for plastic multiplier and tangent matrix are obtained. For implementing the proposed model in finite element code, an implicit method with explicit updating is used for solving the system of nonlinear equations instead of matrix inversion. Corrector relations for stress, back stress and plastic strain tensors as well as damage are introduced. Although, generality has been observed in the damage formulation, the proposed integration scheme is modified to accommodate the Bonora damage model which is implemented in the commercial finite element code MSC.MARC. The numerical implementation is validated by comparing the numerical results with analytical solutions for the damage evolution law under different stress triaxiality levels and damage exponents. Also, two different models are considered for FE simulations and a comparison is made between the uncoupled and the proposed models.

1. Introduction

In metals, damage consists of three different mechanisms including void nucleation, growth and coalescence. It is well known that voids nucleate from hard particles such as inclusion and precipitates. All mechanisms are deeply localized in the material microstructure. Their characteristic size is usually much smaller than the average grain size. Even at rupture, in the region close to the fracture surface, porosity is usually very low and much less than that predicted by the porosity models [1]. As a consequence, the macroscale effects of damage on material behavior are difficult to be evaluated.

The continuum damage mechanics (CDM) based approach used in the study of the ductile damage in metals and alloys was initially proposed by Kachanov [2] and was developed by Lemaitre [3] and Lemaitre and Chaboche [4]. In this framework, damage is one of the state variables which accounts for the progressive degradation due to the irreversible deformation processes occurring at micro-scale in the material. In the anisotropic evolution of damage, various types of damage at the micro-scale level are distributed non-uniformly in the material. Anisotropic damage can be introduced by a forth order tensor [5,6] or more practically by second order damage tensor [7–11]. On the other hand, if voids, cavities and cracks are distributed uniformly in all directions, the material degradation introduces isotropic damage. In this case, damage variable can fully be characterized by a scalar [2,12–14]. Some experimental studies [7,15–17] show that the damage-induced material stiffness degradation reveals its effects by decrease in the elastic modulus and change in Poisson's ratio even when the virgin material is isotropic. This phenomenon cannot be introduced only by the orientation-independent scalar damage models. Therefore, it can be concluded that anisotropic damage theories are more accurate in predicting ductile fracture of materials. However, some researchers neglect the change of Poisson's ratio even in the case of anisotropic damage [11,18]. Some researchers prefer to simplify the damage model by considering a constant Poisson's ratio and assuming an isotropic damage evolution. Literature reviews have shown that this simplification is widely used in the investigations particularly for initially isotropic metal alloys [19–29].

The main purpose of the present work is to consider the true stress instead of the effective stress in the yield function of the constitutive coupled damage-plasticity model. Therefore, a simple scalar form of damage variable and constant Poisson's ratio are assumed to simplify the proposed model and to reduce the

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Nomenclature

b	isotropic hardening exponent	$\mathbf{K}_S$	Residual function for deviatoric stress tensor
f	yield function	$\mathbf{K}_X$	Residual function for backstress tensor
k_1, k_2	kinematic hardening parameters	$\mathbf{N}$	flow direction tensor
n	material hardening exponent	Q	maximum change in yield surface radius
p	total active accumulated plastic strain under multiaxial state of stress	$\hat{R}()$	isotropic hardening function
p^+	active accumulated plastic strain under multiaxial state of stress	$\mathbf{S}$	deviatoric stress tensor
r	associated variable for isotropic hardening	S_0	damage strength energy
A_0	nominal sectional area	$\mathbf{X}$	backstress tensor
A_D	damage sectional area	Y	associated variable for damage (strain energy density release rate)
C_D	corrector for damage	α	damage exponent
C_p	corrector for accumulated plastic strain	$\boldsymbol{\delta}$	Kronker tensor function
$\mathbf{C}_S$	corrector for deviatoric stress tensor	$\boldsymbol{\varepsilon}$	strain tensor
$\mathbf{C}_X$	corrector for backstress tensor	$\boldsymbol{\varepsilon}^e$	elastic strain tensor
D	damage variable	$\boldsymbol{\varepsilon}^p$	plastic strain tensor
D_{cr}	critical damage at failure	$\boldsymbol{\varepsilon}_{cr}$	strain at failure
E_0	Young's modulus for undamaged material	$\boldsymbol{\varepsilon}_{th}$	damage threshold strain (uniaxial)
E	effective (damaged) Young's modulus	λ	plastic multiplier
F_D	damage dissipation potential	ν	Poison's ratio
G_0	shear modulus of undamaged material	$\boldsymbol{\sigma}$	stress tensor
$\mathbf{I}$	fourth order identity tensor	$\tilde{\boldsymbol{\sigma}}$	effective stress tensor
J_2	second deviatoric stress invariant	σ_0	initial yield stress
K_0	bulk modulus for undamaged material	σ_H	hydrostatic stress
K_D	Residual function for damage	$\hat{\sigma}_y()$	radius of yield surface
		$\mathbf{1}$	second order identity tensor

expressions. According to the Kachanov's definition, the damage variable is defined by:

$$D = \frac{A_D}{A_0} \tag{1}$$

When A_D is set equal to zero, the material is in its virgin state represented by $D = 0$. At fracture, we have $A_D = A_0$ and $D = 1$, implying that the entire nominal section is damaged and the material loses its load carrying capability altogether. Effective stress concept, ($\tilde{\boldsymbol{\sigma}} = \boldsymbol{\sigma}/(1-D)$), was initially introduced by Rabotnov [12]. Later and based on the strain equivalence hypothesis [30] the constitutive equations for fully coupled ductile damage with Armstrong–Fredric plasticity model [31] were proposed by Doghri [19]. In fully coupled damage formulations in which the effective stress is considered in both yield function and elastic strain formulation, the damage accumulates as a function of other state variables (usually plastic strain and stress triaxiality). In these formulations, the damage has the following effects:

(a) Damage reduces the elastic properties such as Young modulus but does not affect the Poisson's ratio.
(b) The yield surface is progressively scaled down with the increase of damage. If it is assumed that failure occurs for $D = 1$, then the yield surface eventually reduces to a point. This is usually known as damage softening effect. In FEM application this effect is responsible for the plastic strain localization and mesh effect.

According to statement (a), the damage effects can be observed from the elastic unloading in a plastic deformation process. According to statement (b), during a continuous plastic deformation under strain controlled condition, the damage effects should be observable through the decrease of the actual stress (the stress acting on the net area of material) which is not dependent on geometry changes.

One of the consequences of coupling the damage and the material plastic flow curve is softening which in turn is responsible for plastic strain localization and mesh dependence of the solution. Damage softening is a key feature of porosity models such as Gurson-type based models [32–34].

On the other hand, in partially coupled damage model which is the case in the present study, the true stress is considered instead of the effective stress in the yield function. Therefore, damage affects some, but not all, of the material properties. The level of coupling is based on the experimental evidences of measurable damage effects. This approach eliminates some of the limitations of fully coupled formulations on one hand but introduces a number of computational complexities, on the other hand. Pirondi and Bonora [35] suggested a different level of coupling based on the experimental evidences of ductile damage effects at the macroscopic scale. This approach is known as *partial coupling*, where the damage reduces the material elastic stiffness but it does not appear explicitly in the expression of material plastic flow. As stated above, in previous investigations on damage, the researchers used to consider the conventional tensile stress–strain curve as the flow curve of the virgin material (without damage). In their investigations, the researchers used to take account of damage effects by using effective stress in the yield function. But the fact is that the measured tensile stress–strain curve is influenced by progressive damage in the material and cannot be considered as the flow curve of the virgin material. Therefore, the separation of material flow curve from damage effects may not basically be true. Pirondi and Bonora [35] proposed a model based on the measured flow curve which includes the effects of damage too. The effects of damage are determined through the procedure given by Bonora et al. [36]. In the model proposed by Pirondi and Bonora [35] damage is obtained independently after calculation of the accumulated plastic strain.

Bonora et al. [21] have discussed the difficulties associated with separating the contribution of damage softening from the total plastic strain hardening in tensile flow curve which is obtained from experiment. In particular, they showed that if the geometry changes such as necking or prior failure due to macroscopic crack formation at the center of the sample are correctly accounted for,