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Free convection in power-law fluids from a heated sphere

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ABSTRACT

In this work, the governing field equations describing heat transfer from a heated sphere immersed in quiescent power-law fluids have been solved numerically. In particular, consideration has been given to elucidate the role of Grashof number (Gr), Prandtl number (Pr) and power-law index (n), on the value of the Nusselt number (Nu) for a sphere in the natural convection regime. Further insights are provided by presenting streamline and constant temperature contours. The results presented herein encompass the following ranges of conditions: $10 \le Gr \le 10^7$; $0.72 \le Pr \le 100$ and $0.4 \le n \le 1.8$ thereby covering both shear-thinning and shear-thickening types of fluid behaviours. Broadly, all else being equal, shear-thinning behaviour can enhance the rate of heat transfer by up to three-fold where as shear-thickening can impede it up to $\sim 30-40\%$ with reference to that in Newtonian fluids. The paper is concluded by presenting detailed comparisons with the scant experimental data and the other approximate treatments of this problem available in the literature.

1. Introduction

Heat transfer from a single sphere immersed in moving and stagnant fluids denotes an idealization of numerous industrially important processes. Typical examples include the use of fixed, fluidized and multiphase reactors to carry out several industrially important chemical reactions. Further examples are found in the thermal processing of particulate foodstuffs before canning, and their subsequent heating before use of processed foodstuffs (Carroll et al., 1996). Additional applications are found in the melting of polymer pellets before being extruded or moulded into various shapes. On the other hand, the flow of fluids past a sphere and heat transfer from it also represents a classical model configuration to elucidate the nature of the underlying physical processes thereby adding to our fundamental understanding. Admittedly, most industrial applications entail either clusters of particles and/or non-spherical particles, experience has shown that a thorough understanding of the nature of momentum and heat transfer characteristics of a single sphere often serves as a useful starting point for undertaking the modeling of more realistic systems involving multiple particles and/or non-spherical particles. Consequently, over the years, a wealth of information has accrued on various aspects of flow, heat and mass transfer from a sphere submerged in fluid streams as far as the Newtonian fluids are concerned (Clift et al., 1978; Michaelides, 2006; Polyanin et al., 2002). A cursory examination of these as well as other reviews suggests that heat and mass transfer from a sphere have been studied much less extensively than the corresponding fluid mechanics issues, even in Newtonian fluids.

From a theoretical standpoint, heat transfer always occurs in the mixed regime, i.e., by forced and free convection. The relative importance of the two mechanisms is often ascertained by estimating the value of the so-called Richardson number, defined as, $Ri = Re/Gr^2$ where Re is the Reynolds number based on the bulk flow velocity and Gr is the Grashof number. The value of $Ri \sim 1$ indicates that the bulk (external) velocity is comparable to that induced by the buoyancy effects, and hence it denotes the occurrence of the mixed convection regime where it is not possible to neglect the contribution of free or of the forced convection. Obviously, $Ri \ll 1$ denotes the free convection limit and $Ri \gg 1$ corresponds to the forced convection limit. Suffice it to add here that the bulk of the heat transfer literature relates to the forced convection regime, even for Newtonian fluids. In summary, based on a combination of analytical, numerical and experimental studies, it is possible to estimate the value of heat transfer coefficient from a sphere in stagnant and moving fluids over most conditions of practical interest as far as the Newtonian fluids are concerned (Churchill, 1983; Churchill and Churchill, 1975; Martynenko and Khramtsov, 2005; Westerberg and Finlayson, 1990).

On the other hand, many fluids of high molecular weight (polymer melts and solutions) and of multiphase nature (like suspensions, emulsions, foams, for instance) encountered in scores of industrial settings display more complex flow behaviour than the simple Newtonian fluid behaviour. For instance, most polymeric and multiphase fluids exhibit shear-thinning, shear thickening, visco-elasticity, yield stress, etc. under appropriate conditions. Undoubtedly, the

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commonest of all these is the shear-thinning behaviour wherein the effective viscosity of a fluid decreases with the increasing shear rate. The reverse kind of behaviour seen in concentrated suspensions and pastes is known as shear-thickening (Barnes et al., 1989; Carreau et al., 1997; Chhabra and Richardson, 2008). It is customary to model this type of fluid behaviour using the two-parameter power-law viscosity model. In spite of their wide occurrence in polymer, food, personal care and pharmaceutical products, agricultural chemicals, etc., surprisingly very little is known about the influence of shear-thinning and shear-thickening characteristics on heat transfer from a sphere in such fluids, albeit an extensive body of knowledge does exist on fluid mechanical aspects including drag, wall effects, wake phenomena, etc. (Chhabra, 2006; Dhole et al., 2006a; Song et al., 2009, 2010). The present work is concerned with the influence of power-law flow characteristics on the rate of heat transfer from a heated sphere submerged in stagnant power-law fluids, i.e., in the free convection regime. At the outset, it is, however, instructive to review the currently available studies on free convection from a sphere in Newtonian media and on flow/heat transfer from a sphere in power-law fluids which, in turn, will facilitate the presentation and discussion of the new results obtained in this work.

2. Previous work

Owing to its fundamental significance, considerable research efforts have been expended in studying free convection heat transfer from a heated sphere in Newtonian fluids, typically air or water. Needless to add here that the momentum and energy equations are coupled via the body force term (gravity) and this precludes the possibility of rigorous analytical solutions. Early analytical attempts employ either the boundary layer flow approximation (large values of Grashof number), or rely on the applicability of matched asymptotic expansion method in the other limit of vanishingly small values of Grashof number. Obviously, the boundary layer approximation breaks down in the wake region of the sphere. The utility of boundary layer approach is exemplified by the studies of Merk and Prins (1953–1954), Chiang et al. (1964), Stewart (1971) and Potter and Riley (1980) and others (Jafarpur and Yovanovich, 1992). On the other hand, the small Grashof number case has been tackled amongst others by Fendell (1968), Singh and Hasan (1983), etc. Only over the past 15–20 years, numerical solutions to the complete governing equations have been sought. Geoola and Cornish (1981) were seemingly the first to provide completely numerical solutions for free convection from a sphere in air ($Pr=0.72$) over the range of Grashof number as: $0.05 \le Gr \le 50$. They reported extensive results on drag coefficient and Nusselt number, the latter were shown to be in fair agreement with the experimental results. However, their numerical scheme failed to converge for $Gr > 50$. Subsequently, they (Geoola and Cornish, 1982) extended this work to elucidate the role of Prandtl number on the flow and heat transfer characteristics in the free convection regime by considering additional values of Prandtl number ($Pr=10$ and 100). They circumvented the convergence difficulties by seeking solutions to time-dependent equations. Subsequently, Farouk (1982) reported numerical results on local and mean Nusselt numbers over the range of Rayleigh number ($Ra=Gr \times Pr$) $10^{-1} \le Ra \le 10^4$. Both Fujii et al. (1984) and Riley (1986) provided time-dependent solutions for the free convection regime. For $Pr=0.72$ (air) and $Ra=100$, Fujii et al. (1984) showed that it was sufficient to use the outer boundary of radius of 40 times that of the sphere as far as the value of the steady state average Nusselt number is concerned. They also noted that while their values of the average Nusselt number were within $\pm 2\%$ of those based on the boundary layer approximation, but the two results differ significantly at the stagnation points. Similarly, Riley (1986) solved the time-dependent equations for $Pr=0.72$ and 7 and for $Gr=500$ and 10^4 and he reported the evolution of Nusselt number with time. His results are also consistent with the other predictions available in the literature (Potter and Riley, 1980). Subsequently, Takamatsu et al. (1988) have revisited this flow and elucidated the influence of domain size and Prandtl number on the free convection from a sphere in fluids with $0.7 \le Pr \le 120$. Their results suggest that larger domains are needed with the increasing value of the Rayleigh number in order to obtain the results which are largely free from domain effects. Thus, for $Pr=0.7$ and $Ra=1$, they suggest a spherical domain $(D_\infty/D) \ge 60$ to be adequate. Dudek et al. (1988) have estimated the drag force on a sphere induced by buoyancy currents which were found to be in good agreement with their simulations. Their results, however, relate to extremely small values of the Grashof number, $0.002 \le Gr \le 0.3$ in air. Johnson et al. (1988) also reported numerical results for free convection from a sphere in air which are in line with the previous numerical and experimental results available in the literature. In recent years, there has been a renewed interest in studying free convection from a sphere in Newtonian fluids (Jia and Gogos, 1996a, 1996b; Yang et al., 2007). Jia and Gogos (1996a, 1996b) have reported numerical results on free convection from isothermal spheres in air over extended range of Grashof number, $10 \le Gr \le 10^8$, using both steady and transient simulations. The effect of Prandtl number has been studied extensively by Yang et al. (2007). Hence, excellent analytical and numerical results are available on the momentum and heat transfer characteristics for free convection from a sphere in Newtonian fluids. It is also appropriate to add here that most of these studies are based on the assumption of constant thermo-physical properties, except that for the fluid density which is invariably captured via the well known Boussinesq approximation. The above-mentioned numerical advances have been supplemented by a few experimental studies on free convection from a sphere, e.g., see Amato and Tien (1972), Kranse and Schenk (1965). Most of these have been reviewed by Churchill (1983). Suffice it to say here that based on a combination of analytical, numerical and experimental studies, it is now possible to estimate the value of the mean Nusselt number for a sphere in the free convection regime (Churchill, 1983; Martynenko and Khramtsov, 2005) in Newtonian fluids over most conditions of practical interest.

In contrast, much less work is available on heat transfer from a sphere in power-law fluids. Early studies are based on the boundary layer flow approximations which inevitably entail the assumption of $Gr \to \infty$. The pioneering study of Acrivos (1960) illustrates the utility of this approach. He asserted his results to be applicable for $Pr > 10$. Subsequently, Stewart (1971) has revisited this problem for a range of axisymmetric shapes. As far as known to us, there has been no prior numerical study on free convection heat transfer from a sphere in power-law fluids for finite values of Prandtl and/or of Grashof numbers. Indeed, even the corresponding literature for forced convection from a sphere in power-law fluids is rather limited (Chhabra, 1999, 2006; Dhole et al., 2006b; Kawase and Ulbrecht, 1981).

On the other hand, only Liew and Adelman (1975) have reported experimental results on free convection from a heated sphere in power-law fluids. Over the range of conditions, $0.3 \le n \le 1$ and $8.8 \times 10^5 \le Ra \le 3.4 \times 10^8$, they reported their mean values of the Nusselt number to be in fair agreement with the predictions of Acrivos (1960) and they also put forward an empirical correlation. Amato and Tien (1976) extended their previous study with water to free convection from a heated sphere in power-law fluids using polymer solutions. They presented extensive temperature profiles and Nusselt number data which were broadly in line with the results of Acrivos (1960). By involving the usual analogy between heat and mass transfer, Lee and Donatelli (1989) reported their free convection mass transfer results for spheres to be consistent with the predictions of Acrivos (1960).