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# PI controller design for a class of distributed parameter systems

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ABSTRACT

In this paper, the regulation problem of convection governed dynamical processes described by linear distributed parameter models with position-independent inputs is addressed by combining the method of characteristics (MC) and the Malek-Zavarei and Jamshidi theorem. In this approach, the MC is used to transform the first order partial differential equations model of such processes into a set of time-delayed ordinary differential equations whose elements are used to generate certain matrices that satisfy linear matrix inequalities that render the proportional integral (PI) control parameters that guarantee the process stability. The performance of the proposed PI controller is tested via numerical simulations in the temperature regulation of a linear heat exchanger and compared to a tightly tuned classical PI controller. The application of the proposed controller is extended to regulate the outlet concentration in a nonlinear non-isothermal plug flow reactor. It is shown that the proposed controller is able to handle the infinite dimensional nature of the process model yielding smooth and unsaturated responses around the predetermined set-point trajectory in the face of load changes, time varying boundary conditions and set-point changes.



## 1. Introduction

The control problem for dynamical systems described by distributed parameter models (i.e. models described by Partial Differential Equations (PDE's)) that are governed by diffusive phenomena has been addressed by three approaches (Reinschke and Smith, 2003). In the *early lumping* approach, the controller is built from a high dimensional set of Ordinary Differential Equations (ODE's) that has been derived from the spatial discretization of the original PDE's model. However, this approach has some drawbacks such as the spillover, or failure to stabilize the actual plant (Balas, 1978), and the influence of the spatial discretization method over the observability and controllability properties of the reduced system (Waldraff et al., 1998). Other approaches include the *distributed state-space and semigroup* method and the *infinite dimensional transfer matrix* method which have shown serious difficulties for their practical implementation (Jones and Kerrigan, 2010). For dynamic processes governed by convection, and therefore described by First Order Partial Differential Equations (FOPDE's), the above approaches are no longer applicable (Christofides, 2001). Instead, optimal and geometric control approaches have been used to design distributed feedback controllers for output tracking while guaranteeing the overall stability of the closed-loop system by considering the manipulated input, the controlled output and the measurements as a function of the axial position (Sira-Ramirez, 1989; Christofides and Daoutidis, 1996, 1998a, 1998b; Wu and Liou, 2001; Choi and Lee, 2005; Dubljevic et al., 2005; Chou and Wu, 2007; Vilas et al., 2007) . Other authors have preferred the use of the Method of Characteristics (MC) to transform the FOPDE's model into a set of time-delayed ODE's from which the controller is devised. This was the case of Gundepudi and Friedly (1998) who used such a method for model reduction of a non-isothermal plug flow reactor and a heat exchange process and then, proposed a discrete control scheme for temperature regulation in the first process and the concentration in the second one, by manipulating the flow velocity. Shang et al. (2005) have also used the MC to derive a continuous controller for the output regulation of the heat exchange process studied by Gundepudi and Friedly. More recently, García-Sandoval et al. (2008) used this transformation method to design a robust control scheme that took into account the resulting time delay of the transformed model and showed that only one measurement is needed to yield highly satisfactory behavior in the face of parameter uncertainties, set-point changes and load disturbances. In practice, however, one hardly ever has the flexibility to implement the aforementioned procedures because of their relatively complex structure.

*Abbreviations:* PDE's, partial differential equations; ODE's, ordinary differential equations; FOPDE's, first-order partial differential equations; MC, method of characteristics; LMI's, linear matrix inequalities

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In this paper we addressed the regulation problem of dynamic processes described by linear FOPDE's, by combining the use of the MC and the Malek-Zavarei and Jamshidi theorem (Malek-Zavarei and Jamshidi, 1987) that render a rather simple Proportional-Integral (PI) controller. The MC is used to transform the original linear FOPDE's into a particular set of time delayed ODE's with linear elements that give rise a number of matrices that fulfill the linear matrix inequalities (LMI's) of the aforementioned theorem for some definite positive matrices. Then, the control parameters are calculated by using these last matrices. The proposed control scheme is evaluated via numerical simulations in two study cases: (a) a linear heat exchange process and (b) a nonlinear reacting system taking place in a plug-flow reactor. It is shown that the proposed control schemes were able to yield satisfactory responses in the face of load disturbances, time-varying boundary conditions and set-point changes. The performance of the proposed control in the first study case was further tested when compared to a tightly tuned classical PI controller.

The paper is organized as follows. In Section 2, the considered process model is introduced, the control problem is defined and basic concepts related to the MC are introduced. The design of the PI controller is described in Section 3 whereas simulation results are shown in Section 4. Finally, some concluding remarks are established in Section 5.

## 2. Problem formulation

### 2.1. The process model

Let us consider a spatially distributed process with plug flow dynamics which is described by the following linear FOPDE's:

$$\frac{\partial X(z,t)}{\partial t} = -v\frac{\partial X(z,t)}{\partial z} + AX(z,t) + Bu(t) + \Gamma d(t) \tag{1}$$

$$y(t) = CX(L,t) \tag{2}$$

with the initial and boundary conditions

$$X(z,0) = \alpha(z) \tag{3}$$

$$X(0,t) = \beta(t) \tag{4}$$

where $X(z,t) \in \mathcal{H}^n[(0,L),\mathbb{R}^n]$ is the state vector whereas $y(t) \in \mathbb{R}^r$, $u(t) \in \mathbb{R}^m$ and $d(t) \in \mathbb{R}^p$ denote, respectively, the output, the inputs and disturbances of the system and are assumed fully independent of the axial position. Here, $z \in [0,L] \subset \mathbb{R}$ and $t \in [0,\infty)$ denote the axial position and time, respectively. $\alpha(z) \in \mathcal{H}^n[(0,L),\mathbb{R}^n]$ and $\beta(t) \in \mathbb{R}^n$ are vectorial functions with well-defined first order derivatives (i.e. they are at least $C^1$ vectorial functions). $\mathcal{H}^n[(0,L),\mathbb{R}^n]$ is the infinite dimensional Hilbert space of $n$-dimensional-like vectorial functions defined on the interval $[0,L]$ where $L$ is the length of the system while $v \subset \mathbb{R}^+$ is the convective velocity which is assumed constant. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{r\times n}$ and $\Gamma \in \mathbb{R}^{n\times p}$ are constant matrices.

### 2.2. Control problem statement

The *boundary control problem for spatially distributed convective processes with position-independent inputs* is defined as the problem of finding a dynamic controller that, for a constant reference $y_r$, guarantees that the output error, defined as $e(t) \equiv y(t) - y_r$, is asymptotically stable.

### 2.3. The method of characteristics

The FOPDE's (1) describe waves traveling with constant velocity $v$ in the positive $z$-direction. As reported by Rhee et al. (2000), the MC provides a precise framework to solve this type of equations. Thus, if the MC is applied to (1), one can find a solution for the initial and boundary conditions (3) and (4) (see Appendix A):

$$X(z,t) = \begin{cases} e^{At}\alpha(z-vt) + \eta(t,t) & \forall z > vt \\ e^{(Az/v)}\beta\left(t-\frac{z}{v}\right) + \eta\left(t,\frac{z}{v}\right) & \forall z \le vt \end{cases} \tag{5,6}$$

where $\eta(t,\delta) = \int_{t-\delta}^{t} e^{A(t-\lambda)}(Bu(\lambda) + \Gamma d(\lambda))d\lambda$. At the fixed point $z=L$, the solution is given by

$$X(L,t) = \begin{cases} e^{At}\alpha(v(\tau-t)) + \eta(t,t) & \forall t < \tau \\ e^{A\tau}\beta(t-\tau) + \eta(t,\tau) & \forall t \ge \tau \end{cases} \tag{7,8}$$

where $\tau = L/v$ denotes the residence time (or time delay). Now, let us define $x_1(t) = X(L,t)$ and $x_2(t) = X(vt,t)$ when $t < \tau$ and $x_2(t) = X(L,t)$ when $t \ge \tau$. Then, by taking the derivative of (5), (7) and (8) with respect to time and by setting $z = vt$, one finally gets the dynamics of $x_1(t)$ and $x_2(t)$

**Subsystem 1** $\forall t < \tau$

$$\begin{aligned} \dot{x}_1(t) &= Ax_1(t) + Bu(t) + \Gamma d(t) + \omega(t) \\ \dot{x}_2(t) &= Ax_2(t) + Bu(t) + \Gamma d(t) \\ X(L,t) &= x_1(t) \end{aligned} \tag{9}$$

**Subsystem 2** $t \ge \tau$

$$\begin{aligned} \dot{x}_1(t) &= 0 \\ \dot{x}_2(t) &= Ax_2(t) + Bu(t) + \Gamma d(t) + \omega(t) \\ &\quad - e^{A\tau}Bu(t-\tau) - e^{A\tau}\Gamma d(t-\tau) \\ X(L,t) &= x_2(t) \end{aligned} \tag{10}$$

with $x_1(0) = \alpha(L)$ and $x_2(0) = \beta(0)$ as the initial conditions

$$\omega(t) = \begin{cases} -ve^{At}\frac{d\alpha(z)}{dz}\Big|_{v(t-\tau)} & \forall t < \tau \\ e^{A\tau}\frac{d\beta(t)}{dt}\Big|_{t-\tau} + Ae^{A\tau}\beta(t-\tau) & \forall t \ge \tau \end{cases}$$

**Remark 1.** Note that the solution for the considered distributed parameter model consists of two parts: The first one is given by Eq. (5) that describes the behavior of the variable $X$ in the region $(z > vt)$ where the "fresh" fluid that enters at $z=0$ has not yet reached the outlet and therefore, this region is only affected by the initial condition. The other part of the solution is Eq. (6) that describes the behavior of $X$ in the zone $(z \le vt)$ where the initial fluid is no longer in the system, and as a consequence, this zone is only influenced by the boundary condition. These regions are shown in Fig. 1.

**Remark 2.** $x_2(t)$ in subsystem (9) denotes the time evolution of an element ("fresh" fluid) initially located at $z=0$ that is influenced by the boundary condition and flows through the process

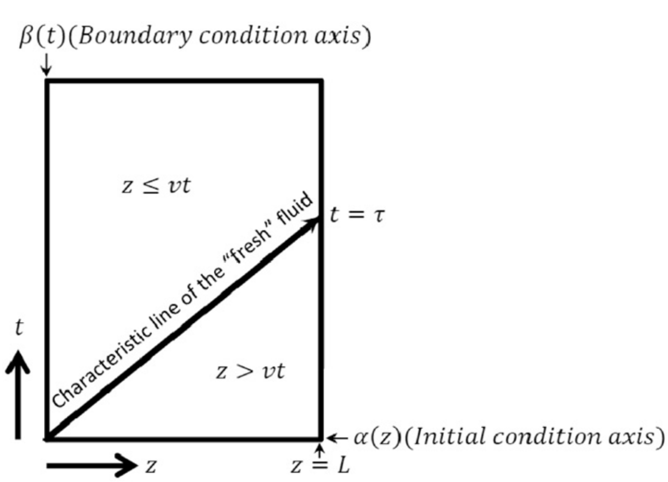


Fig. 1. Regions arising in the solution for the considered distributed parameter model.