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# An incremental damage theory for micropolar composites taking account of progressive debonding and particle size effect

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ABSTRACT

The paper presents an incremental damage model for determining the overall mechanical behaviors of particulate-reinforced micropolar composites. The particle size effect and progressive-debonding damage are fully accounted for within the present framework, which makes use of the micropolar Eshelby's tensor for characterizing the size-dependent constraint of the matrix microstructure on the particles, a critical stress criterion for the interfacial debonding, and Qiu-Weng's energy approach, together with Hu's variation method for the equivalent stress of the matrix. Finally, the present model was applied for the predictions of the overall stress–strain relations of the composites that reinforced by either particles or voids, and are subjected to the uniaxial tension. It is shown that all the predictions are in good agreement with the experiments and finite element computations.



## 1. Introduction

Particle size is being widely regarded as an important factor significantly influencing the effective properties of the composites. The particle size is smaller, yield and flow stresses are more enhanced, while fracture toughness and ductility are often more reduced [1–3]. In addition, particle size also greatly affects the evolution of debonding damage during the deformation. On the development of high-performance composites containing nanoparticles, it is important to make clear the particle size effect and the damage progress in particle-reinforced composites. This contribution is to develop an analytical model taking account of the particle size effect as well as the damage evolution.

Generally, two schemes were adopted to study the influence of progressive damage on the stress–strain relation of the composites. One was finite element method (FEM) analysis based on a unit cell [4–10], and the other was the micromechanics-based method. Tohgo et al. [11,12] proposed an incremental damage model of particle-reinforced composites taking account of plasticity of the matrix and progressive debonding damage of particles based on the Eshelby's equivalent inclusion method [13] and Mori–Tanaka's mean field concept [14]. Ju and Lee [15] and Sun et al. [16] regarded the partially-debonded particles in the composites as fictitious orthotropic inclusions without debonding according to Zhao and Weng [17], and proposed a progressive debonding damage model of particle-reinforced composites based on the ensemble-volume averaging procedure. FEM analyses based on a unit cell are seriously influenced by the periodical boundary conditions imposed on the representative volume element (RVE). Therefore, micromechanics-based methods are more physically reasonable and much more accurate as compared to those FEM simulations based on a unit cell.

Until now, many studies have been performed on the particle size effect. Niordson and Tvergaard [18] and Xue et al. [19] carried out a unit-cell based FEM analysis for discontinuously reinforced composites by using strain gradient plasticity [20], and discussed the size effects of reinforcements on the overall deformation behaviors of the composites. Nan and Clarke [21] extended the effective medium approach by introducing the particle size effect into the stress–strain relation of *in situ* matrix in the composites and damage criterion of particles. Liu and Sun [22] and Jiang et al. [23,24] developed analytical models of particle-reinforced composites by introducing an interphase between particles and matrix and discussed the particle size effect on the deformation due to an interphase. Tohgo et al. [25] incorporated the effect of dislocation density into the *in situ* yield stress of the matrix and then studied the damage behaviors of metal matrix composite (MMC) on the basis of incremental damage theory. Based on the micropolar Eshelby's solutions for spherical and cylindrical inclusions by Cheng and He [26], Sharma and Dasgupta [27] extended the Mori–Tanaka's method to estimate the effective elastic behaviors of the micropolar composites.

To predict the non-linear behaviors of micropolar composites, Liu and Hu [28] and Xun et al. [29] presented analytical micromechanical methods by making use of the classical secant moduli

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scheme with second-order stress moment [30]. Although the elasto-plastic behaviors of particle-reinforced composites have been well studied in many published works, the progressive debonding damage was not carefully considered yet. Based on the micropolar elasticity theory, the main objective of the paper is supposed to propose the exact formulas of incremental damage theory to analyze not only the particle size effect but also the debonding damage.

In this paper, the incremental damage model by Tohgo et al. [11,12] is reformulated on the basis of micropolar elasticity theory. Numerical studies of the stress–strain relations under uniaxial tension are performed for the composites with a certain size distribution of particles. Subsequently, the effects of debonding damage, particle size and particle volume fraction on the stress–strain response of the composites are discussed.

## 2. Micropolar elasticity theory

Since the constituents of the composites are assumed micropolar materials, the micropolar elasticity theory is briefly explained. In absence of body forces and body moment, the governing equations for determining elastic stress and couple stress of a Centro-symmetric and isotropic micropolar material are given by

$$\varepsilon_{ij} = u_{j,i} - e_{kij}\phi_k, \quad k_{ij} = \phi_{j,i} \tag{1a}$$

$$\sigma_{ij,i} = 0, \quad m_{ij,i} + e_{jik}\sigma_{ik} = 0 \tag{1b}$$

$$\sigma_{ij} = \delta_{ij}\lambda\varepsilon_{kk} + (\mu+\kappa)\varepsilon_{ij} + (\mu-\kappa)\varepsilon_{ji} \tag{1c}$$

$$m_{ij} = \delta_{ij}\alpha k_{kk} + (\beta+\gamma)k_{ij} + (\beta-\gamma)k_{ji} \tag{1d}$$

where $\sigma_{ij}$ and $m_{ij}$ denote the non-symmetric stress and couple stress tensors, $\varepsilon_{ij}$ and $k_{ij}$ non-symmetric strain and torsion tensors, $u_i$ and $\varphi_i$ displacement and micro-rotation vectors, respectively, $e_{ijk}$ permutation tensor and $\delta_{ij}$ kronecker delta. $\lambda$ and $\mu$ are the classical Lame's constants and $\kappa$, $\gamma$, $\beta$, $\alpha$ the new elastic constants in micropolar theory. $\lambda$, $\mu$ and $\kappa$ have dimensions of [N/m$^2$], and $\gamma$, $\beta$ and $\alpha$ have dimensions of [N]. The boundary conditions applied on the boundary $\partial\Omega$ of volume $\Omega$ are given as,

$$\sigma_{ij}n_j = t_i, \quad m_{ij}n_j = p_i \quad \text{on } \partial\Omega \tag{2a}$$

$$u_i = \bar{u}_i, \quad \phi_i = \bar{\phi}_i \quad \text{on } \partial\Omega \tag{2b}$$

where $\bar{u}_i$, $\bar{\phi}_i$, $t_i$ and $p_i$ are the uniform displacement, micro-rotation, constant surface force and couple stress separately, and $n_j$ the outer normal unit tensor on $\partial\Omega$.

Three intrinsic characteristic lengths of $l_1$, $l_2$ and $l_3$ are introduced to account for the dimensional difference between the two sets of moduli, and are defined by,

$$l_1 = \sqrt{\gamma/\mu}, \quad l_2 = \sqrt{\beta/\mu}, \quad l_3 = \sqrt{\alpha/\mu} \tag{3}$$

For the sake of simplicity, three characteristic lengths are assumed to be equal ($l_1 = l_2 = l_3 = l$). The constitutive equations are written in the compact form when we denote $\sigma'_{(ij)}$, $\sigma_{\langle ij\rangle}$, $\sigma_{kk}$ and $\varepsilon'_{(ij)}$, $\varepsilon_{\langle ij\rangle}$, $\varepsilon_{kk}$ as the deviatoric symmetric, anti-symmetric and hydrostatic parts of the stress and strain tensors, respectively (similar notations for the couple-stress and torsion tensors):

$$\sigma'_{(ij)} = 2\mu\varepsilon'_{(ij)}, \quad \sigma_{\langle ij\rangle} = 2\kappa\varepsilon_{\langle ij\rangle}, \quad \sigma_{kk} = 3K\varepsilon_{kk} \tag{4a}$$

$$m'_{(ij)} = 2\beta k'_{(ij)}, \quad m_{\langle ij\rangle} = 2\gamma k_{\langle ij\rangle}, \quad m_{kk} = 3Nk_{kk} \tag{4b}$$

with

$$K = \lambda + \frac{2}{3}\mu, \quad N = \alpha + \frac{2}{3}\beta \tag{5}$$

where subscripts $(\cdot)$ and $\langle\cdot\rangle$ denote the symmetric and anti-symmetric parts of the tensor, respectively.

Similar to the classical Cauchy material, a $J_2$-type effective equivalent stress is established for the micropolar material by,

$$\sigma_{eq}^2 = \frac{3}{2}\left[\sigma'_{(ij)}\sigma'_{(ij)} + l^2\left(m'_{(ij)}m'_{(ij)} + m_{\langle ij\rangle}m_{\langle ij\rangle}\right)\right] \tag{6}$$

## 3. Eshelby's tensors and Mori–Tanaka's method for a spherical inclusion

According to Cheng and He's conclusions [26], the perturbed strain $\varepsilon_{ij}^{pt}(\mathbf{x})$ and torsion $k_{ij}^{pt}(\mathbf{x})$ at the material point $\mathbf{x}$ due to an inhomogeneity are expressed in terms of the Eigenstrain $\varepsilon_{ij}^*(\mathbf{x})$ and Eigentorsion $k_{ij}^*(\mathbf{x})$ as,

$$\varepsilon_{ij}^{pt}(\mathbf{x}) = S_{ijkl}(\mathbf{x})\varepsilon_{kl}^*(\mathbf{x}) + \Gamma_{ijkl}(\mathbf{x})k_{kl}^*(\mathbf{x}) \tag{7a}$$

$$k_{ij}^{pt}(\mathbf{x}) = \hat{S}_{ijkl}(\mathbf{x})\varepsilon_{kl}^*(\mathbf{x}) + \widehat{\Gamma}_{ijkl}(\mathbf{x})k_{kl}^*(\mathbf{x}) \tag{7b}$$

where $S_{ijkl}$, $\Gamma_{ijkl}$, $\widehat{S}_{ijkl}$ and $\widehat{\Gamma}_{ijkl}$ are the micropolar Eshelby's tensors. The most attractive conclusion in the classical Eshelby's theory is that the strain (and so the stress) is uniform inside an inclusion, but which does not hold for the micropolar medium any more. For the sake of simplicity and convenience in the application, Sharma recommend adopting the average values of micropolar Eshelby's tensors over the inclusion instead of the original forms [27]. Subsequently, such the strategy was widely used later and confirmed by Liu and Hu [28], Sarvestani [31] and Berveiller [32], etc.

Liu and Hu [28] computed the volume-averaged values of micropolar Eshelby's tensors over an inclusion and confirmed that $\langle\Gamma_{ijkl}\rangle = \langle\widehat{S}_{ijkl}\rangle = 0$, and the specific formulas of $\langle S_{ijkl}\rangle$ and $\langle\widehat{\Gamma}_{ijkl}\rangle$ are expressed as,

$$\langle S_{ijkl}\rangle = T_1\delta_{ij}\delta_{kl} + (T_2+T_3)\delta_{ik}\delta_{jl} + (T_2-T_3)\delta_{il}\delta_{jk} \tag{8a}$$

$$\langle\widehat{\Gamma}_{ijkl}\rangle = Q_1\delta_{ij}\delta_{kl} + (Q_2+Q_3)\delta_{ik}\delta_{jl} + (Q_2-Q_3)\delta_{il}\delta_{jk} \tag{8b}$$

where

$$T_1 = \frac{3\lambda-2\mu}{15(\lambda+2\mu)} + \frac{4g(d+g)\kappa}{5d^3(\kappa+2\mu)}\Gamma(g)$$

$$T_2 = \frac{3\lambda+8\mu}{15(\lambda+2\mu)} - \frac{6g(d+g)\kappa}{5d^3(\kappa+2\mu)}\Gamma(g)$$

$$T_3 = \frac{4\kappa+3\mu}{6\mu} - \frac{2g(d+g)(\kappa+\mu)^2}{d^3\mu(\kappa+2\mu)}\Gamma(g) - \frac{h(d+h)}{2d^3}\Gamma(h)$$

$$Q_1 = -(\gamma+\beta)\frac{(2\mu+\kappa)(g+d)}{20\mu\kappa g d^3}\Gamma(g) + (5\alpha+\gamma+\beta)\frac{(h+d)}{10\kappa h d^3}\Gamma(h)$$

$$Q_2 = \frac{3(\beta+\gamma)(2\mu+\kappa)(g+d)}{40\mu\kappa g d^3}\Gamma(g) + (\gamma+\beta)\frac{(h+d)}{10\kappa h d^3}\Gamma(h)$$

$$Q_3 = \frac{(\gamma-\beta)(2\mu+\kappa)(g+d)}{8\mu\kappa g d^3}\Gamma(g)$$

$$\Gamma(y) = e^{-d/y}[d\cosh(d/y) - y\sinh(d/y)]$$

$$h^2 = \frac{\alpha+\beta+\gamma}{2\kappa}, \quad g^2 = \frac{(2\mu+\kappa)\gamma}{4\mu\kappa}$$

where $\delta_{ij}$ is the Kronecker-delta function, $d$ the particle diameter, and $h$ and $g$ have dimensions of [m]. $C_{ijkl}$ and $D_{ijkl}$ denote the elasticity tensors of the micropolar medium, and are given by,