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Homogenization and averaging methods to predict elastic properties of pre-impregnated composite materials

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ABSTRACT

The paper presents a homogenization method and some averaging methods to predict the elastic properties of multiphase pre-impregnated composite materials like Sheet Molding Compounds (SMCs). The upper and lower limits of the homogenized coefficients for a 27% fibers volume fraction SMC have been computed. A comparison between the upper and lower limits of the homogenized elastic coefficients for a 27% fibers volume fraction SMC and the experimental data is presented. The estimation model used as a homogenization method of these heterogeneous composite materials, gave emphasis to a good agreement between this theoretical approach and experimental data.

1. Introduction

The objectives of the paper is to compute the upper and lower limits of the homogenized elastic coefficients for a common 27% fibers volume fraction SMC based on a homogenization method described by Ene and Pasa [1] as well as some averaging methods of the Young and shear moduli of various SMCs with different fibers volume fractions.

The most obvious mechanical model which features a multiphase composite material is a pre-impregnated material, known as prepreg. In the wide range of prepregs the most common used are Sheet and Bulk Molding Compounds. A Sheet Molding Compound (SMC) is a pre-impregnated material, chemically thickened, manufactured as a continuous mat of chopped glass fibers, resin (known as matrix), filler and additives, from which blanks can be cut and placed into a press for hot press molding. The result of this combination of chemical compounds is a heterogeneous, anisotropic composite material, reinforced with discontinuous fibers [2,3].

A typical SMC material is composed of the following chemical compounds: calcium carbonate, chopped glass fibers roving, unsaturated polyester resin, low-shrink additive, styrene, different additives, pigmented paste, release agent, magnesium oxide paste, organic peroxide and inhibitors in various volume fractions. The matrix system plays a significant role within a SMC, acting as compounds binder and being "embedded material" for the reinforcement. To decrease the shrinkage during the cure of a SMC prepreg, filler have to be added in order to improve the flow capabilities and the uniform fibers transport in the mold. For materials that contain many compounds, an authentic, general method of dimensioning is difficult to find.

In a succession of hypotheses, some authors tried to describe the elastic properties of SMCs based on ply models and on material compounds [4-6]. The glass fibers represent the basic element of SMC prepreg reinforcement. The quantity and roving orientation determine, in a decisive manner, the subsequent profile of the SMC structure's properties. The following information is essential for the development of any model to describe the composite materials behavior: the thermo-elastic properties of every single compound and the volume fraction concentration of each compound [7]. Theoretical researches regarding the behavior of heterogeneous materials lead to the elaboration of some homogenization methods that try to replace a heterogeneous media with a homogeneous one [8-12]. The aim is to obtain a computing model, which takes into account the microstructure or the local heterogeneity of a material.

2. A homogenization method

We consider a domain Ω from R^3 space, in x_i coordinates, domain considered a SMC composite material, in which a so-called

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replacement matrix (resin and filler) represents the field Y_1 and the reinforcement occupies the field Y_2 seen as a bundle of glass fibers.

Let us consider the following equation [1]:

$$f(x) = -\frac{\partial}{\partial x_i}\left[a_{ij}(x) \cdot \frac{\partial u}{\partial x_j}\right]; \quad a_{ij} = a_{ji} \tag{1}$$

alternatively, written under the equivalent form:

$$f = -\frac{\partial p_i}{\partial x_i}; \quad p_i = a_{ij} \cdot \frac{\partial u}{\partial x_j}. \tag{2}$$

In the case of SMC materials that present a periodic structure containing inclusions, $a_{ij}(x)$ is a function of x. If the period's dimensions are small in comparison with the dimensions of the whole domain then the solution u of the Eq. (1) can be equal with the solution suitable for a homogenized material, where the coefficients a_{ij} are constants. In the R^3 space of y_i coordinates, a parallelepiped with y_i^0 sides is considered, as well as parallelepipeds obtained by translation $n_i y_i^0$ (n_i integer) in axes directions. The functions [1]:

$$a_{ij}^{\eta}(x) = a_{ij}\left(\frac{x}{\eta}\right), \tag{3}$$

can be defined, where η is a real, positive parameter. Notice that the functions $a_{ij}(x)$ are ηY-periodical in variable x (ηY being the parallelepiped with ηy_i^0 sides). If the function $f(x)$ is in Ω defined, the problem at limit is [1]:

$$f(x) = -\frac{\partial}{\partial x_i}\left[a_{ij}^{\eta}(x) \cdot \frac{\partial u^{\eta}}{\partial x_j}\right], \quad u^{\eta}|_{\partial\Omega} = 0. \tag{4}$$

Similar with Eq. (2), the vector $\vec{p}^{\eta}$ defines the following elements [1]:

$$p_i^{\eta}(x) = a_{ij}^{\eta}(x) \cdot \frac{\partial u^{\eta}}{\partial x_j}. \tag{5}$$

For the function $u^{\eta}(x)$, an asymptotic development will be looking for, under the form [1]:

$$u^{\eta}(x) = u^0(x,y) + \eta^1 u^1(x,y) + \eta^2 u^2(x,y) + \cdots; \quad y = \frac{x}{\eta}, \tag{6}$$

where $u^i(x, y)$ are elements, Y-periodical in y variable. The derivatives of the functions $u^i(x, y)$ behave in the following manner [1]:

$$\frac{d}{dx_i} \to \frac{\partial}{\partial x_i} + \frac{1}{\eta} \cdot \frac{\partial}{\partial y_i}. \tag{7}$$

If the values of $u^i\left(x, \frac{x}{\eta}\right)$ are compared in two homologous points P_1 and P_2, homologous through periodicity in neighbour periods, it can be notice that the dependence in $\frac{x}{\eta}$ is the same and the dependence in x is almost the same since the distance P_1P_2 is small. Let us consider P_3 a point homologous to P_1 through periodicity, situated far from P_1. The dependence of u^i in y is the same but the dependence in x is very different since P_1 and P_3 are far away. For instance, in the case of two points P_1 and P_4 situated in the same period, the dependence in x is almost the same since P_1 and P_4 are very close, but the dependence in y is very different since P_1 and P_4 are not homologous through periodicity. The function u^{η} depends on the periodic coefficients a_{ij}, on the function $f(x)$ and on the boundary $\partial\Omega$. The development (6) is valid at the inner of the boundary $\partial\Omega$, where the periodic phenomena are prevalent. Using the development (6), the expressions $\frac{\partial u^{\eta}}{\partial x_i}$ and p^{η} are [1]:

$$\begin{aligned}\frac{\partial u^{\eta}}{\partial x_i} &= \left(\frac{\partial}{\partial x_i} + \frac{1}{\eta} \cdot \frac{\partial}{\partial y_i}\right) \cdot (u^0 + \eta \cdot u^1 + \cdots) \\ &= \frac{\partial u^0}{\partial x_i} + \frac{\partial u^1}{\partial y_i} + \eta \cdot \left(\frac{\partial u^1}{\partial x_i} + \frac{\partial u^2}{\partial y_i}\right) + \cdots,\end{aligned} \tag{8}$$

$$p_i^{\eta}(x) = p_i^0(x,y) + \eta \cdot p_i^1(x,y) + \eta \cdot p_i^2(x,y) + \cdots, \tag{9}$$

where:

$$p_i^0(x,y) = a_{ij}(y) \cdot \left(\frac{\partial u^0}{\partial x_j} + \frac{\partial u^1}{\partial y_j}\right), \quad p_i^1(x,y) = a_{ij}(y) \cdot \left(\frac{\partial u^1}{\partial x_j} + \frac{\partial u^2}{\partial y_j}\right), \ldots \tag{10}$$

represent the homogenized coefficients.

3. Application for a 27% fibers volume fraction Sheet Molding Compound

In the case of a SMC composite material which behaves macroscopically as a homogeneous elastic environment, is important the knowledge of the elastic coefficients. Unfortunately, a precise calculus of the homogenized coefficients can be achieved only in two cases: the one-dimensional case and the case in which the matrix- and inclusion coefficients are functions of only one variable. For a SMC material is preferable to estimate these homogenized coefficients between an upper and a lower limit.

Since the fibers volume fraction of common SMCs is 27%, to lighten the calculus, an ellipsoidal inclusion of area 0.27 situated in a square of side 1 is considered. The plane problem will be considered and the homogenized coefficients will be 1 in matrix and 10 in the ellipsoidal inclusion. In Fig. 1 the structure's periodicity cell of a SMC composite material is presented, where the fibers bundle is seen as an ellipsoidal inclusion.

Let us consider the function $f(x_1, x_2) = 10$ in inclusion and 1 in matrix. To determine the upper and the lower limit of the homogenized coefficients, first the arithmetic mean as a function of x_2-axis followed by the harmonic mean as a function of x_1-axis must be computed. The lower limit is obtained computing first the harmonic mean as a function of x_1-axis and then the arithmetic mean as a function of x_2-axis. If we denote with $\varphi(x_1)$ the arithmetic mean against x_2-axis of the function $f(x_1, x_2)$, it follows:

$$\varphi(x_1) = \int_{-0.5}^{0.5} f(x_1, x_2)dx_2 = 1, \text{ for } x_1 \in (-0.5; -0.45) \cup (0.45; 0.5), \tag{11}$$

$$\varphi(x_1) = \int_{-0.5}^{0.5} f(x_1, x_2)dx_2 = 1 + 9.45\sqrt{0.2025 - x_1^2}, \quad \text{for } x_1 \in (-0.45;\ 0.45). \tag{12}$$

The upper limit is obtained computing the harmonic mean of the function $\varphi(x_1)$:

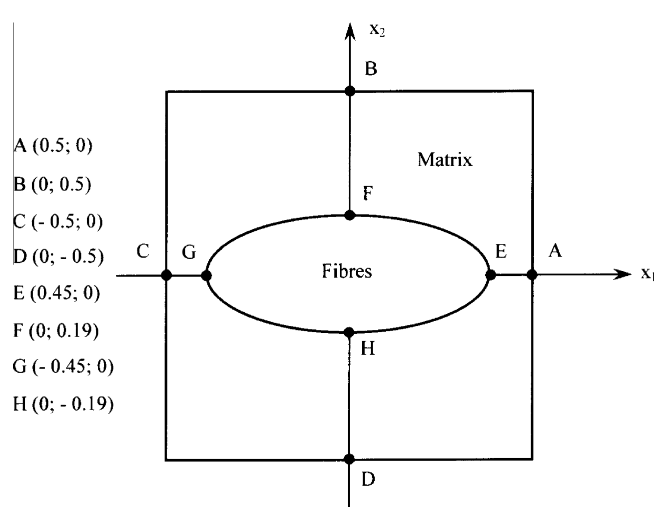

Fig. 1. Structure's periodicity cell of a SMC material with 27% fibers volume fraction. The points A–H are given with their coordinates.