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Periodically distributed parallel cracks in a functionally graded piezoelectric (FGP) strip bonded to a FGP substrate under static electromechanical load ☆

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ABSTRACT

In this paper, the Fourier integral transform–singular integral equation method is presented for the problem of a periodic array of cracks in a functionally graded piezoelectric strip bonded to a different functionally graded piezoelectric material. The properties of two materials, such as elastic modulus, piezoelectric constant and dielectric constant, are assumed in exponential forms and vary along the crack direction. The crack surface condition is assumed to be electrically impermeable or permeable. The mixed boundary value problem is reduced to a singular integral equation over crack by applying the Fourier transform and the singular integral equation is solved numerically by using the Lobatto–Chebyshev integration technique. The analytic expressions of the stress intensity factors and the electric displacement intensity factors are derived. The effects of the loading parameter λ, material constants and the geometry parameters on the stress intensity factor, the energy release ratio and the energy density factor are studied.

1. Introduction

Piezoelectric materials (PMs) have been widely used as a smart material in electromechanical devices due to the demand of transform from mechanical to electrical loadings, and vice versa. In recent years the new trends in material design for a smart structure seem to be toward bonding metallic or composite materials to the piezoelectric elements. As a result, interface de-bonding may happen in bonded non-homogeneous materials. To improve the reliability and durability problems arising largely from high residual and thermal stress, poor interfacial bonding strength, the functionally graded piezoelectric materials (FGPMs) as a new class of advanced composites have been developed. The stress peaks will be induced at the interfaces to cause failure such as cracking or de-bonding.

In the aspect of fracture mechanics applications in non-homogeneous materials, assuming an exponentially spatial variation of the elastic modulus, Erdogan and Wu [1] and Delale and Erdogan [2] have provided a series of analytic solutions for cracks in non-homogeneous elastic solids under mechanical loading. Up to now, a number of studies have been performed for the cracked piezoelectric materials [3–8]. But, in engineering practice, one certain crack is rare, cracks generally arise agminate. Some of the agminate cracks can be considered ideally as periodic cracks. Among these works, Li and Li [9] and Li and Wu [10] studied the problem of periodic array of cracks in the isotropic and anisotropic material. Several scholars considered the fracture problem of periodic cracks in the homogeneous piezoelectric medium or along the interface of bimaterials [11,12].

Recently, some researchers start to investigate the fracture behavior in FGPMs. The material properties may vary along the crack line or normal to the crack surface. Wang and Node [13] firstly studied the thermopiezoelectric fracture problem of a functionally graded piezoelectric layer bonded to a metal. They obtained the thermal flow, stress and electric displacement intensity factors and predict the direction of crack extension by using the energy density theory. Li and Weng [14] solved the problem of a functionally graded piezoelectric material (FGPM) strip containing a finite crack normal to boundary surfaces. The elastic stiffness, piezoelectric constant and the dielectric permittivity vary continuously along the thickness. It is found that the singular behaviors of both stress and electric displacement are the same as those in a homogeneous piezoelectric material (PM). In this crack problem, an increase in the gradient of the material properties will reduce the magnitude of the intensity factors. This finding makes us to research the problem of FGPMs by using the methods which investigates the problem of piezoelectric materials. Wang [15] considered the mode III crack problem in FGPM, where the material properties are assumed in a class of functional form such that an analytic solution is possible. Hu et al. [16] studied the problem of a crack located in a functionally gradient piezoelectric interlayer between two dissimilar homogeneous piezoelectric half-planes. Recently, the fracture behavior of a weak discontinuous interface between

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two piezoelectric strips under electromechanical loads is investigated by Li and Lee, using the methods of Fourier integral transform and Cauchy singular integral equation [17].

Although a variety of challenging issues related to certain crack problems in the functionally graded piezoelectric materials have been addressed, one of the remaining problems that need to fully understood is that of a periodic array of cracks in such media. To date, Erdogan and Ozturk [18] investigated the anti-plane problem of periodic cracks in functionally graded coatings. Wang and Mai [19] studied the problem of a periodic array of cracks in an infinite functionally graded material under mechanical and thermal loading. Chen and Liu [20] treated the dynamic anti-plane problem for a functionally graded piezoelectric strip containing a periodic array of parallel cracks. Recently, Ding and Li [21] studied the problem of periodic interface cracks in a functionally graded coating-substrate structure. However, to the authors' knowledge, few papers considered the solution for the problem of periodic cracks in FGPM.

In this paper, we use the Fourier transform–singular integral equation method to investigate the problem of a FGPM strip containing a periodic array of parallel cracks bonded to a different functionally graded piezoelectric material. First, this crack problem can be reduced into a system of singular integral equations over crack after applying the Fourier transform, and the singular integral equation are then solved numerically by using Lobatto–Chebyshev integration technique. Numerical results are presented to demonstrated that the proposed method is efficient for the periodic crack problem in bonded FGPMs.

2. Formulation of the problem

Fig. 1 shows a FGPM strip perfectly bonded to a different functionally graded piezoelectric material along y-axis. The FGPM strip is assumed to contain periodic cracks perpendicular to the interface. The crack of length and the x-coordinate of the crack center are defined as $2a_0 = b - a$ and $d = (b + a)/2$, respectively. Since the poling directions of piezoelectric materials are orientated along z-axis, the antiplane mechanical field and inplane electric field are coupled. The constitutive equation can be written as

$$\tau_{xzk} = c_{44k}(x)\frac{\partial w_k}{\partial x} + e_{15k}(x)\frac{\partial \phi_k}{\partial x}, \quad \tau_{yzk} = c_{44k}(x)\frac{\partial w_k}{\partial y} + e_{15k}(x)\frac{\partial \phi_k}{\partial y},$$
$$D_{xk} = e_{15k}(x)\frac{\partial w_k}{\partial x} - \varepsilon_{11k}(x)\frac{\partial \phi_k}{\partial x}, \quad D_{yk} = e_{15k}(x)\frac{\partial w_k}{\partial y} - \varepsilon_{11k}(x)\frac{\partial \phi_k}{\partial y}, \tag{1}$$

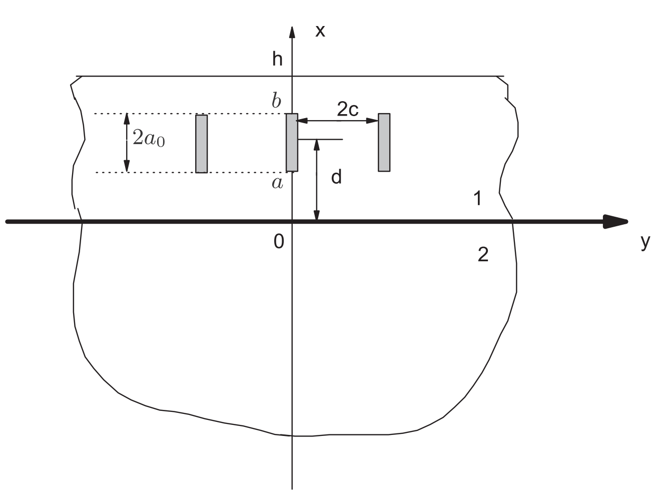

Fig. 1. The geometry of periodically cracked FGPM strip bonded to a different FGPM.

where τ_{izk}, w_k, D_{ik} and $\phi_k (i = x, y, k = 1, 2)$ are the shear stresses, antiplane displacements, inplane electrical displacements and electric potentials, respectively. The variations of material constants $c_{44k}(x)$, $e_{15k}(x)$ and $\varepsilon_{11k}(x)$ called the shear modulus, piezoelectric constants, and dielectric constants, respectively, are assumed in the following exponential forms:

$$c_{441}(x) = c_0 e^{\beta x}, \quad e_{151}(x) = e_0 e^{\beta x}, \quad \varepsilon_{111}(x) = \varepsilon_0 e^{\beta x}, \quad 0 < x < h,$$
$$c_{442}(x) = c_0 e^{\gamma x}, \quad e_{152}(x) = e_0 e^{\gamma x}, \quad \varepsilon_{112}(x) = \varepsilon_0 e^{\gamma x}, \quad x < 0, \tag{2}$$

where β and γ is called non-homogeneous parameters. The constants c_0, e_0 and ε_0 are the material properties at interface.

The static equilibrium equation and Maxwell's equation under electro-static condition are given as

$$\frac{\partial \tau_{xzk}}{\partial x} + \frac{\partial \tau_{yzk}}{\partial y} = 0, \quad \frac{\partial D_{xk}}{\partial x} + \frac{\partial D_{yk}}{\partial y} = 0, \quad k = 1, 2. \tag{3}$$

Substituting Eq. (1) into Eq. (3) and using the relation (2), we obtain the following equations:

$$\nabla^2 w_1 + \beta\frac{\partial w_1}{\partial x} = 0, \quad \nabla^2 \psi_1 + \beta\frac{\partial \psi_1}{\partial x} = 0,$$
$$\phi_1(x, y) = \frac{e_0}{\varepsilon_0} w_1(x, y) + \psi_1(x, y), \tag{4}$$

the governing equations of functionally graded piezoelectric materials may be expressed as

$$\nabla^2 w_2 + \gamma\frac{\partial w_2}{\partial x} = 0, \quad \nabla^2 \psi_2 + \gamma\frac{\partial \psi_2}{\partial x} = 0,$$
$$\phi_2(x, y) = \frac{e_0}{\varepsilon_0} w_2(x, y) + \psi_2(x, y), \tag{5}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator.

By separating the homogeneous solution through an appropriate superposition, the problem may be reduced to a perturbation solution in which self-equilibration crack surface tractions are the only nonzero external loads. Since periodicity, only the problem for $0 < y < c$ is considered. Together with the mixed boundary conditions as follows:

$$w_1(0, y) = w_2(0, y), \quad \phi_1(0, y) = \phi_2(0, y),$$
$$\tau_{xz1}(0, y) = \tau_{xz2}(0, y), \quad D_{x1}(0, y) = D_{x2}(0, y),$$
$$\tau_{xz1}(h, y) = 0, \quad D_{x1}(h, y) = 0,$$
$$w_2(x, 0) = 0, \quad \phi_2(x, 0) = 0, \quad -\infty \leqslant x < 0,$$
$$w_1(x, c) = 0, \quad \phi_1(x, c) = 0, \quad 0 < x < h,$$
$$w_2(x, c) = 0, \quad \phi_2(x, c) = 0, \quad -\infty \leqslant x < 0, \tag{6}$$

and the boundary conditions on the crack surfaces can be written as

$$w_1(x, 0) = 0, \quad \phi_1(x, 0) = 0, \quad 0 < x < a, \quad b < x < h,$$
$$\tau_{yz1}(x, 0) = -\tau(x), \quad D_{y1}(x, 0) = -D(x), \quad a < x < b, \tag{7}$$

for the impermeable case, and

$$w_1(x, 0) = 0, \quad 0 < x < a, \quad b < x < h,$$
$$\phi_1(x, 0) = 0, \quad 0 < x < h,$$
$$\tau_{yz1}(x, 0) = -\tau(x), \quad D_{y1}(x, 0) = -D_c(x, 0) = -D(x), \quad a < x < b, \tag{8}$$

for the permeable case, where $D_c(x, 0)$ denotes the electric displacement of the space of the crack itself.

3. Singular integral equations

We proceed with the electrically impermeable case. By using Fourier transform method, we can obtain