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Formulation of a unit cell of a reduced size for plain weave textile composites

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ABSTRACT

A unit cell of a much reduced size for plain weave textile composites has been formulated in this paper. Use has been made all available and independent types of geometric symmetries, including translations, reflections and rotations. These geometric symmetries have been interpreted into appropriate mechanical terms based on the concept of relative displacements, from which boundary conditions for the unit cell have been derived systematically. All loading conditions in terms of macroscopic stresses/strains have been considered for full characterisation of such composites. As loading conditions associate with shear have to be derived individually and separately from the direct stress loading conditions due to the use of reflectional and rotational symmetries, the boundary conditions obtained may therefore appear to be tedious but comprehensive and systematic. The unit cell formulated has been validated and an example of application has been illustrated.

1. Introduction

Plain weave is one of the most common types of fabrics used in production of textile composites. A large number of micromechanical FE analyses on such composites can be found in the literature using unit cells of various shapes and sizes [1–6], to name but a few. Geometric periodicity alone defines a unit cell as shown in Fig. 1 (within the white frame including the coloured patches), referred to as *full model* hereafter in this paper. To formulate the full mode as a unit cell for micromechanical analyses, use has to be made of the translational symmetries and the boundary conditions obtained are often called *periodic conditions* [7–10]. Without challenging the well established terminology, 'periodic conditions' could be a source of confusion in formulation of unit cells. While periodicity is present for the geometry and stress and strain fields, it is absent for the displacement field. Unfortunately, when FE is employed for the analysis, it is the displacement field that is of prime significance as strains and stresses are subsequently derived from displacements and, more importantly, boundary conditions for the unit cell have to be prescribed in terms of displacements.

Within the full model as described above, there are apparently reflectional symmetries available. Once utilised properly, the size of the unit cell can be reduced to a quarter, as shown in blue sketch (including the red block) in Fig. 1, referred to as the *quarter model* hereafter in this paper. Boundary conditions can be obtained relatively easily from these symmetries and appropriate formulation can be found in [11].

The size can be further reduced to a quarter again if one makes use of rotational symmetries as present in the quarter model. There are at least two useful independent ones, i.e. two rotations of 180° about the two centrelines of the quarter model in the plane of the fabric. Subsequently, after the use of the two available and independent rotational symmetries, a unit cell as sketched in red[1] in Fig. 1 is obtained, which is of a size of 1/16 of the original full model. It will be referred to as the *unit cell* throughout this paper. Efforts will be made to have this unit cell fully formulated.

While such a unit cell of drastically reduced size is attractive, its formulation is anything but trivial. In [4], the analysis was carried out on some assumed stress/strain fields, boundary conditions for the unit cell were not actually requires. Ref. [6] assumed that opposite faces in the x and y directions remain parallel after deformation under in-plane direct loading which is not generally true. While under in-plane shear loading, approximate nature of the boundary conditions adopted was admitted. The account as can be found in [3] is most relevant to the present study. All the geometric symmetries employed in the above discussion have been identified there. However, the formulation of a unit cell consists of two parts: (i) identifying geometric symmetries and (ii) interpreting these symmetries into appropriate mechanical boundary conditions for the unit cell. The present paper claims its novelty on the second part. Boundary conditions for such a unit cell were presented in [3], which might look similar to what are presented in the present paper. However, seemingly subtle but theoretically and practically fundamental differences are present, which will

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[1] For interpretation of colour in Figs. 1–3 and 5–9, the reader is referred to the web version of this article.

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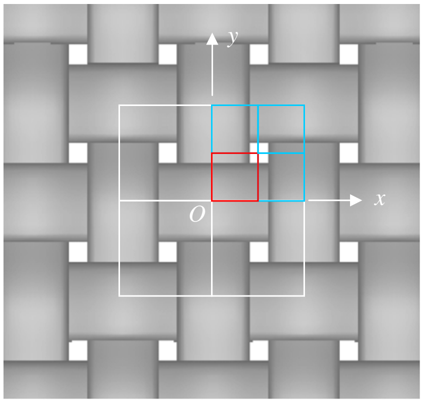

Fig. 1. Plain weave and unit cell models based on the use of various symmetries.

be discussed as it develops later in this paper. Providing appropriate boundary conditions to such a unit cell in a completely rational way for a plain weave textile composite is the objective of the present paper, which has not been available in the literature, given the presence of [3]. The boundary conditions will be derived rigorously first and validated. No part of the formulation will rest on any unjustified intuition. Applications will be made to demonstrate the use of such a unit cell.

In the literature, boundary conditions for unit cells have been proposed intuitively, more often than not, unjustified. Rather casual treatments can be found in many sources. Luckily enough, they sometimes work under a special condition, e.g. a special type of loading condition, such as uniaxial tension/compression. However, when they are confronted with more general conditions, such as shear loading, the chances are that most of them will fall apart. This is almost certainly one of the reasons why very few accounts can be found in the literature which touches upon shear as a loading condition.

If one wishes to take full advantage of the symmetries present in the plain weave pattern, different types of symmetries will have to be involved, including translations, reflections and rotations. It will be shown that the complexity will be beyond what one could possibly handle based on his/her intuition, although there is no formidable mathematical difficulties, provided that the derivation is guided by clear conscience of geometrical and mechanical implications of each operation involved.

2. Boundary conditions for the full and quarter models

The boundary conditions for the full and quarter models have been provided in [10,11]. For the ease of reference in the subsequent development in this paper, they have being listed in this section. In this paper, an assumption will be made that the plain weave textile composite concerned represents a layer of a laminated block where a large number of such layers are present and aligned properly so that a translational symmetry in the thickness direction (z) can therefore be utilised.

Assume that the weft and warp spacings are b_x and b_y, respectively, and the thickness of the layer of the composite is $2b_z$. The full model is then defined in the domain

$$-b_x \leqslant x \leqslant b_x, \quad -b_y \leqslant y \leqslant b_y \quad \text{and} \quad -b_z \leqslant z \leqslant b_z \tag{1a}$$

while the quarter model is defined in the domain

$$0 \leqslant x \leqslant b_x, \quad 0 \leqslant y \leqslant b_y \quad \text{and} \quad -b_z \leqslant z \leqslant b_z \tag{1b}$$

The unit cell is reduced into the following domain:

$$0 \leqslant x \leqslant b_x/2, \quad 0 \leqslant y \leqslant b_y/2 \quad \text{and} \quad -b_z \leqslant z \leqslant b_z \tag{1c}$$

2.1. Full model

The boundary conditions for the full model resulting from translational symmetries are given as follows [10]:

$$\begin{cases} u|_{x=b_x} - u|_{x=-b_x} = 2b_x\varepsilon_x^0 \\ v|_{x=b_x} - v|_{x=-b_x} = 0 \\ w|_{x=b_x} - w|_{x=-b_x} = 0 \end{cases} \quad \text{for faces} \perp x\text{-axis} \tag{2a}$$

$$\begin{cases} u|_{y=b_y} - u|_{y=-b_y} = 2b_y\gamma_{xy}^0 \\ v|_{y=b_y} - v|_{y=-b_y} = 2b_y\varepsilon_y^0 \\ w|_{y=b_y} - w|_{y=-b_y} = 0 \end{cases} \quad \text{for faces} \perp y\text{-axis} \tag{2b}$$

$$\begin{cases} u|_{z=b_z} - u|_{z=-b_z} = 2b_z\gamma_{xz}^0 \\ v|_{z=b_z} - v|_{z=-b_z} = 2b_z\gamma_{yz}^0 \\ w|_{z=b_z} - w|_{z=-b_z} = 2b_z\varepsilon_z^0 \end{cases} \quad \text{for faces} \perp z\text{-axis} \tag{2c}$$

2.2. Quarter model

After applying the reflectional symmetries about the planes $x = 0$ and $y = 0$, referred to hereafter as x-plane (perpendicular to the x-axis) and y-plane (perpendicular to the y-axis), respectively, the boundary conditions for the quarter model are obtained as follows [11]. Be reminded that they vary from one loading condition to another.

Under $\sigma_x^0,\ \sigma_y^0,\ \sigma_z^0$ or any combination of them, which is a symmetric loading condition under both reflectional symmetries about the x- and y-planes:

$$\begin{aligned} & u|_{x=0} = 0, \quad u|_{x=b_x} = b_x\varepsilon_x^0 \\ & v|_{y=0} = 0, \quad v|_{y=b_y} = b_y\varepsilon_y^0 \\ & u|_{z=b_z} - u|_{z=-b_z} = 0, \quad v|_{z=b_z} - v|_{z=-b_z} = 0 \quad \text{and} \\ & w|_{z=b_z} - w|_{z=-b_z} = 2b_z\varepsilon_z^0 \end{aligned} \tag{3a}$$

Under τ_{yz}^0, which is symmetric under the reflectional symmetry about the x-plane but antisymmetric about the y-plane:

$$\begin{aligned} & u|_{x=0} = 0, \quad u|_{x=b_x} = 0 \\ & u|_{y=0} = w|_{y=0} = 0, \quad u|_{y=b_y} = w|_{y=b_y} = 0 \\ & u|_{z=b_z} - u|_{z=-b_z} = 0, \quad v|_{z=b_z} - v|_{z=-b_z} = 2b_z\gamma_{yz}^0 \quad \text{and} \\ & w|_{z=b_z} - w|_{z=-b_z} = 0 \end{aligned} \tag{3b}$$

Under τ_{xz}^0, which is antisymmetric under the reflectional symmetry about the x-plane but symmetric about the y-plane:

$$\begin{aligned} & v|_{x=0} = w|_{x=0} = 0, \quad v|_{x=b_x} = w|_{x=b_x} = 0 \\ & v|_{y=0} = 0, \quad v|_{y=b_y} = 0 \\ & u|_{z=b_z} - u|_{z=-b_z} = 2b_z\gamma_{xz}^0, \quad v|_{z=b_z} - v|_{z=-b_z} = 0 \quad \text{and} \\ & w|_{z=b_z} - w|_{z=-b_z} = 0 \end{aligned} \tag{3c}$$

Under τ_{xy}^0, which is antisymmetric under both reflectional symmetries about the x- and y-planes:

$$\begin{aligned} & v|_{x=0} = w|_{x=0} = 0, \quad v|_{x=b_x} = w|_{x=b_x} = 0 \\ & u|_{y=0} = w|_{y=0} = 0, \quad u|_{y=b_y} = b_y\gamma_{xy}^0, \quad w|_{y=b_y} = 0 \\ & u|_{z=b_z} - u|_{z=-b_z} = 0, \quad v|_{z=b_z} - v|_{z=-b_z} = 0 \quad \text{and} \\ & w|_{z=b_z} - w|_{z=-b_z} = 0 \end{aligned} \tag{3d}$$

Having used the reflectional symmetries above, a combination of above four loading conditions is forbidden because of different natures of the symmetries, i.e. symmetric and antisymmetric. This is a trade-off for the reduction in the size of the model to be