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Finite element modeling for bending and vibration analysis of laminated and sandwich composite plates based on higher-order theory

Tran Minh Tu [a,*], Le Ngoc Thach [a], Tran Huu Quoc [b]

[a] University of Civil Engineering, 55 Giai Phong Road, Hanoi, Viet Nam
[b] Hanoi University of Technology, 1 Dai Co Viet Road, Hanoi, Viet Nam

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ABSTRACT

A nine-nodded rectangular element with nine degrees of freedom at each node is developed for the bending and vibration analysis of laminated and sandwich composite plates. The theory accounts for parabolic distribution of the transverse shear strains through thickness of the plate and rotary inertia effects. The parametric effects of plate: aspect ratio, angle of fiber orientation, side-to-thickness ratio and degree of orthotropy on in-plane stresses, transverse shearing stresses, displacements, and fundamental frequencies are shown. Results are compared with existing numerical solutions.

1. Introduction

The use of sandwich structures in marine, transport, civil construction, aerospace application is growing rapidly due to their outstanding characteristics like high resistance to fatigue failure induced by acoustic pressure, high strength and stiffness for the given weight, etc. The first departure from classical plate theory (CLPT) was introduced by Reissner, who derived his first-order plate theory for homogenous isotropic plates in equilibrium using an assumed stress approach. Mindlin extended this theory to the elastodynamic analysis of plates using a displacement-based first-order that also includes rotary inertia effects.

Since the pioneering works of Reissner and Mindlin, numerous first-, second- and higher-order plate theories have been proposed. In general, these theories are based on either equivalent single-layer or layerwise assumptions utilizing displacement-based, stress-based, and mixed formulations. In single-layer theory, the displacement components represent the weighted average through the thickness of the sandwich plates.

The first-order shear deformation theory (FSDT) first developed by Yang et al. [1] gives a state of constant shear strain through the thickness of the plate. However, according to three dimensional (3D) elasticity theory, the shear strain varies at least quadratically through the thickness. The so-called shear correction factors were introduced to correct the discrepancy in the shear forces of FSDT and 3D elasticity theories. The value of shear correction factors depend on the constituent ply properties, lamination scheme, geometry and boundary conditions.

Higher-order deformation plate theories (HSDT) are those in which the displacement are expanded to quadratic or higher powers of the thickness co-ordinate. Of this higher-order theories, the one proposed by Reddy and co-worker [2] was the first to obtain the equilibrium equations in a consistent manner using the principle of virtual displacements. Lewinski investigated the mathematical aspects of Reddy's theory, while Rohwer [3] showed that the higher-order theory of Reddy is one of the best considering various parameters.

The finite element approach has proved to be a powerful and widely applicable method for the bending and vibration analysis of complex problems for which analytical solutions are nearly impossible to solve [4–6]. Numerous finite element models of the various theories have been put forward over the year. Finite element solutions for laminated sandwich plates were presented by Khatua and Cheung [7] using triangular and rectangular element which permit freedom of in-plane displacements. Based on FSDT, the Mindlin plate element are essentially based on Reissner–Mindlin assumption with application to both thick and thin plates. However, a defect of this class of elements was detected when thin plates were analyzed. For the HSDT, Phan and Reddy [8] developed displacement-based finite element models based on Reddy's special third-order theory used C^1 interpolation functions for the transverse displacement and C^0 interpolation functions for the other displacements.

* Corresponding author. Tel.: +84 479871723.
E-mail address: tpnt2002@yahoo.com (T.M. Tu).

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In this paper the simple refined higher-order displacement theory of Reddy [9] with a C^0 finite element formulation (the continuity of the unknown only had to occur between elements) is used to study bending and vibration analysis of laminated composite and sandwich plates. The accuracy of results of original computer program, using MATLAB programming language will be verified through existing examples from the literature.

2. Theory and formulation

Consider a laminated plate composed of n orthotropic lamina. The integer k denotes the layer number that starts from the plate bottom (Fig. 1). The displacement field is assumed in the following form [9].

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\theta_x(x,y,t) + z^2 u_0^*(x,y,t) + z^3\theta_x^*(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) + z\theta_y(x,y,t) + z^2 v_0^*(x,y,t) + z^3\theta_y^*(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{1}$$

In these equations u, v, w are the displacement components of a generic point in plate space in the x-, y-, z-directions, respectively; u_0, v_0 are in-plane displacements of a point laying in the middle plane, and θ_x, θ_y are the normal rotations about the y and x axes respectively. The parameters $u_0^*, v_0^*, \theta_x^*, \theta_y^*$ are higher-order terms in the Taylor's series expansion.

By substituting Eq. (1) into the general linear strain–displacement relations, the following relations are obtained

$$\begin{aligned} \varepsilon_x &= \varepsilon_{x0} + zk_x + z^2\varepsilon_{x0}^* + z^3k_x^*; \\ \varepsilon_y &= \varepsilon_{y0} + z\cdot k_y + z^2\varepsilon_{y0}^* + z^3k_y^*; \\ \gamma_{xy} &= \gamma_{xy0} + zk_{xy} + z^2\gamma_{xy0}^* + z^3k_{xy}^*; \\ \gamma_{xz} &= \gamma_{xz0} + zk_{xz} + z^2\gamma_{xz0}^*; \\ \gamma_{yz} &= \gamma_{yz0} + zk_{yz} + z^2\gamma_{yz0}^* \end{aligned} \tag{2}$$

The constitutive relations for a typical orthotropic kth lamina with laminate co-ordinate axes (x, y, z) can be written as

$$\{\sigma\}_k = \{\, \sigma_x \quad \sigma_y \quad \sigma_{xy} \quad \sigma_{xz} \quad \sigma_{yz} \,\}_k^T = \{\overline{Q}\}_k \{\, \varepsilon_x \quad \varepsilon_y \quad \gamma_{xy} \quad \gamma_{xz} \quad \gamma_{yz} \,\}_k^T \tag{3}$$

where the terms of $\{\overline{Q}\}_k$ matrix of layer k are referred to the laminate axes and can obtained from the $\{Q\}_k$ corresponding to the fiber orientations with appropriate transformation, as outlined in literature [10] (the superscript T refers the transpose of a matrix/vector)

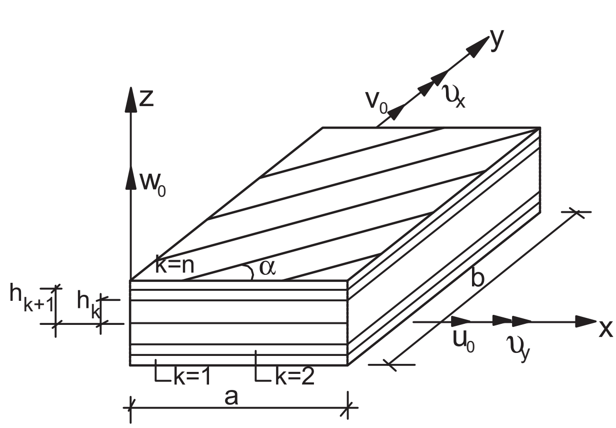

Fig. 1. A typical sandwich plate geometry with laminate reference axes, and fiber orientation.

3. Finite element model

In the present work a C^0 nine-node quadrilateral finite element with nine degrees of freedom per node is used. The displacement vector $\{d\}$ structures of any point on the mid-plane is given by

$$d = \sum_{i=1}^{9} N_i(\xi,\eta)d_i \tag{4}$$

where $d_i = \{\, u_{0i} \quad v_{01} \quad w_{0i} \quad \theta_{xi} \quad \theta_{yi} \quad u_{0i}^* \quad v_{0i}^* \quad \theta_{xi}^* \quad \theta_{yi}^* \,\}$ is displacement vector corresponding to node i, $N_i(\xi, \eta)$ are the interpolating or shape functions associated with node i. ξ, η are the natural co-ordinates.

With the generalized displacement vector $\{d\}$ known at all point within the element, the generalized strain vector at any point given by

$$\varepsilon = \sum_{i=1}^{9} B_i d_i \tag{5}$$

where B_i is a differential operator matrix of shape function.

The governing differential equations of motion can be derived using Hamilton's principle

$$\delta\int_{t_1}^{t_2}(U - T + A)dt = 0 \tag{6}$$

where t is the time, U is total strain energy of the system, T is the total kinetic energy of the system and A is the work done by external forces.

Using the standard finite element procedure, the governing differential equations of motion can be rewritten as

$$[M^e]\{\ddot{d}\} + [K^e]\{d\} = \{F^e\} \tag{7}$$

where the element stiffness matrix:

$$[K^e] = \int\int [B]^T[D][B]dx\,dy \tag{8}$$

the element mass matrix

$$[M] = \int\int [N]^T[m][N]dx\,dy \tag{9}$$

with

$$[m] = \begin{bmatrix} I_0 & 0 & 0 & I_1 & 0 & I_2 & 0 & I_3 & 0 \\ 0 & I_0 & 0 & 0 & I_1 & 0 & I_2 & 0 & I_3 \\ 0 & 0 & I_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I_1 & 0 & 0 & I_2 & 0 & I_3 & 0 & I_4 & 0 \\ 0 & I_1 & 0 & 0 & I_2 & 0 & I_3 & 0 & I_4 \\ I_2 & 0 & 0 & I_3 & 0 & I_4 & 0 & I_5 & 0 \\ 0 & I_2 & 0 & 0 & I_3 & 0 & I_4 & 0 & I_5 \\ I_3 & 0 & 0 & I_4 & 0 & I_5 & 0 & I_6 & 0 \\ 0 & I_3 & 0 & 0 & I_4 & 0 & I_5 & 0 & I_6 \end{bmatrix} \tag{10}$$

where $I_i = \sum_{l=1}^{n}\sum_{h_l}^{h_{l+1}} z^i\rho_l\,dz, i = 0, 1, \ldots, 6$

and the nodal load vector:

$$\{F^e\} = \int\int [N]^T q\,dx\,dy \tag{11}$$

After evaluating the stiffness and mass matrices for all elements, they have been assembled together to form the overall stiffness and mass matrix $[K]$, $[M]$ of the structure. We obtain the system equation as following:

$$[M]\{\ddot{q}\} + [K]\{q\} = \{F\} \tag{12}$$

From the system equation, let $\{F\} = 0$, we have the equation of the free vibration problem:

$$[M]\{\ddot{q}\} + [K]\{q\} = 0 \tag{13}$$