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Dimensional analysis of the thermomechanical problem arising during through-hardening of cylindrical steel components

Caner Şimşir *, Thomas Lübben, Franz Hoffmann, Hans-Werner Zoch

Foundation Institute of Materials Science (IWT), Badgasteiner St. 3, 28359 Bremen, Germany

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ABSTRACT

This article concerns the dimensional analysis of the complex thermomechanical problem arising during through-hardening of cylindrical steel components. A complete and independent set of physically based dimensionless numbers was derived using the weak formulation method and the individual effects of dimensionless numbers with sensitive material parameters were investigated by finite element simulations. The results demonstrate potential of the approach in simplification of the model and the identification of physically relevant dimensionless combinations.

1. Introduction

Isotropic dimensional changes during through-hardening of steel workpieces are unavoidable due to density difference between the initial and final microstructures. Beside this, shape changes also occur due to inhomogeneous plastic deformation during the process. In contrast to transformation free quenching which necessitates overrun of a critical Biot number (Bi^p) above which yielding can occur [1]; anisotropy in dimensional changes are theoretically unavoidable after through-hardening due to transformation-induced plasticity (TRIP) which occurs without a yield limit [2].

The prediction of quenching distortion is a challenging task that requires solution of a complicated thermo-elastoplastic model. Currently, the widely used solution technique is the numerical solution of the problem that relies on time-consuming computer simulations. Accordingly, there is a practical interest in simplification of the problem using analytical methods. Dimensional analysis is one of the possible candidates for this purpose. In contrast to the diversity of its applications in fluid dynamics, dimensional analysis is not so frequently applied in solid mechanics, especially in thermo-elastoplasticity. Some applications to elastic shells and to turning process of steel components can be found in [3] and [4], respectively.

Dimensional analysis allows simplification of the model and identification of the physically relevant dimensionless combinations independent from the individual parameters [5,6]. In addition, proper scaling exhibits its advantage in numerical simulations since the results are directly comparable. Moreover, scaling experiments can be performed on prototypes and questions of similarity can be dealt with dimensional analysis [7]. For example, prototyping can be a favorable tool in the future to predict the distortion of large components such as shafts and bearings races used in ships. For such components, the removal of the heat treatment distortion by hard machining is extremely expensive and may be impossible for some cases.

Former studies demonstrated the potential of dimensional analysis in the investigation of heat treatment distortion. For example, early studies of Frerichs et al. [8] and Wolff et al. [1] clarified many aspects of dimensional analysis of transformation free quenching process, while Landek et al. [9] proposed a technique for approximate prediction of heat treatment distortion using the dimensionless quantities. Finally, Wolff et al. [10] suggested an extension of the model applicable to through-hardening and Simsir et al. [11] illustrated the significance of Biot number with a discussion of general problems and workarounds for the proposed extension.

In this study, the former studies [10,11] were extended by investigation of the effect of four dimensionless numbers. The selection was based on sensitivity analysis to ensure the significance of investigated dimensionless numbers. Sensitivity analysis revealed the martensite-start temperature (M_s), Koistinen–Marburger parameter (M_o), thermal expansion coefficient (α) and the transformation strain (ε^{pt}) as highly sensitive parameters. The dimensionless numbers containing those parameters were nominated as the "martensite-start number" ($\prod_3$), "Koistinen–Marburger number" ($\prod_4$),

* Corresponding author. Tel.: +49 421 218 53 39; fax: +49 421 218 53 33.
E-mail address: simsir@iwt-bremen.de (C. Şimşir).

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"thermal strain number" ($\prod_5$) and "transformation strain number" ($\prod_6$), respectively. Then, the investigations were performed using computer simulations in which the investigated dimensionless number was varied independently while the remaining were kept at their reference values. The results were presented as functions of Biot number ($\prod_1$).

The rest of the paper is organized as follows: In Section 2, the underlying mathematical model is summarized with a discussion of possible modifications and critical Biot number (Bi*) concept. In Section 3, numerical procedure for the simulations, variation strategy for dimensionless numbers and evaluation of results is presented. In Section 4, the results are discussed and elaborated considering possible physical mechanisms. Finally, the study is concluded with an outlook of the research in Section 5.

2. Mathematical model

2.1. Dimensional analysis

The distortion of cylindrical components such as shafts, discs and rings after quench hardening can be described by a mathematical model with 30 parameters that spans over four independent dimensions – mass ($\mathscr{M}$), length ($\mathscr{L}$), time ($\mathscr{T}$), temperature ($\boldsymbol{\Theta}$) [12]. Table 1 summarizes the involved parameters with their corresponding dimensions. On this table, it should be noted that all material properties are phase dependent and individual parameters for each phase are not written explicitly for the sake of brevity. Henceforth, the properties of austenite and martensite phases are differentiated in the text using "*a*" and "*m*" subscripts, respectively.

According to Buckingham's $\prod$ theorem [7], the mathematical model can be reduced to a dimensionless equivalent governed by 26 dimensionless numbers. The determination of complete and independent set of dimensionless numbers can be performed by purely algebraic method – i.e. Buckingham's approach – or weak formulation of the problem in terms of dimensionless quantities [5]. In [10], Wolff et al. presented an application of the former method. However, the physical relevance of dimensionless number set derived by purely algebraic method may be obscure since the choice of a dimensionless number set from infinite number of possible sets is performed heuristically. On the other hand, the physical relevance of dimensionless numbers derived using the weak formulation method is ensured since the dimensionless numbers originate from the governing equations.

In this study, an independent and complete set of dimensionless numbers governing the through-hardening distortion was derived using the weak formulation method. Table 2 illustrates this set with nomination and categorization of dimensionless numbers based on their appearance in governing equations and their physical interpretations.

The set was calculated according to following procedure:

(1) The problem is simplified according to the following assumptions:
   - The temperature dependence of material parameters was neglected by using suitable average values. The major consequence of this assumption is the limitation of the quantitative application of the method to a limited range of Biot numbers. The details of the averaging method and its consequences can be found elsewhere [11].
   - The phase dependence of E, ν, n, c and α was neglected in formulation for simplicity. However, based on former experience, the ratio of those parameters for different phases (E_m/E_a, n_m/n_a, etc.), which can be considered as additional dimensionless numbers, was kept constant during variations. The validity of this approach is justified in Section 4.3.
   - Through-hardening must be achieved, i.e., no phase transformation other than martensitic transformation should occur in the system. For the investigated steel, this requires that no bainitic phase transformation occur before martensitic transformation. In other words, the Biot number should be higher than "the critical Biot number (Bi*)". Further information about the critical Biot number can be found in Section 2.2.

(2) Although the following points are not important for theoretical justification of the model on conceptual basis, they should be considered additionally for practical usage:
   - In order to avoid the additional dimensionless numbers governing heating and austenization process, the reference (initial) and final state for the problem was set to pure austenite at the austenization temperature (T_o) and martensite + retained austenite mixture at ambient temperature (T_∞). However, such a reference state is not feasible for real life distortion problems where the natural reference state is the initial microstructure (spheroidized carbides in a ferritic matrix at room temperature) at RT. A convenient remedy for this problem is to keep the thermo-metallurgical strain at T_o with respect to initial microstructure at RT constant. This necessitates to keep $\rho_a(T_o)$ constant, but the variations in dimensionless numbers can be done using the other parameters. Consequently, this additional constraint makes the use of $L(20\,^\circ\mathrm{C})$, $D(20\,^\circ\mathrm{C})$ equivalent to $L(T_o)$, $D(T_o)$ in the definition of the dimensionless numbers.

Table 1
List of model parameters with their corresponding dimensions.

		$\mathscr{M}$	$\mathscr{L}$	$\mathscr{T}$	$\boldsymbol{\Theta}$			$\mathscr{M}$	$\mathscr{L}$	$\mathscr{T}$	$\boldsymbol{\Theta}$
Geometry						*Transformation*					
L	Length	0	1	0	0	ΔH	Transformation enthalpy	0	2	−2	0
D	Outer diameter	0	1	0	0	M_s	Martensite start temperature	0	0	0	1
$D_{i,b}$	Inner Diameter (bottom)	0	1	0	0	M_o	Koistinen–Marburger Temp.	0	0	0	1
$D_{i,t}$	Inner diameter (top)	0	1	0	0	κ	TRIP constant	−1	−1	2	0
Process						*Thermomechanical*					
T_0	Austenization temperature	0	0	0	1	α	Thermal expansion coefficient	0	0	0	−1
T_∞	Ambient temperature	0	0	0	1	E	Elastic modulus	1	1	−2	0
h	Heat transfer coefficient	−1	3	−3	−1	ν	Poisson's ratio	0	0	0	0
Thermal						K	Ramberg–Osgood coefficient	1	1	−2	0
ρ	Density	1	−3	0	0						
c	Specific heat capacity	0	−2	−2	−1	n	Ramberg–Osgood exponent	0	0	0	0
λ	Thermal conductivity	−1	2	−3	−1	σ^o	Yield strength	1	1	−2	0