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Asymptotic homogenisation in linear elasticity. Part II: Finite element procedures and multiscale applications

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A B S T R A C T

The asymptotic expansion homogenisation (AEH) method can be used to solve problems involving physical phenomena on continuous media with periodic microstructures. In particular, the AEH is a useful technique to study of the behaviour of structural components built with composite materials. The main advantages of this approach lie on the fact that (i) it allows a significant reduction of the problem size and (ii) it has the capability to characterise stress and deformation microfields. In fact, specific equations can be developed to define these fields, in a process designated by localisation and not found on typical homogenisation methods. In the AEH methodology, overall material properties can be derived from the mechanical behaviour of selected periodic microscale representative volumes (also known as representative unit-cells, RUC). Nevertheless, unit-cell based modelling requires the control of some parameters, such as reinforcement volume fraction, geometry and distribution within the matrix material. The need for variety and flexibility leads to the development of automatic geometry generation algorithms. Additionally, the unstructured finite element meshes required by these RUC are usually non-periodic and involve the control of specific periodic boundary conditions. This work presents some numerical procedures developed in order to support finite element AEH implementations, rendering them more efficient and less user-dependent. The authors also present a numerical study of the influence of the reinforcement volume fraction on the overall material properties for a metal matrix composite (MMC) reinforced with spherical ceramic particles. A general multiscale application is shown, with both the homogenisation and localisation procedures.

1. Introduction

When numerically simulating the behaviour of metal matrix composite (MMC) materials reinforced with ceramic particles it is often necessary to use non-structured finite element meshes. The need for significant resources in terms of memory and CPU time leads to the use of dedicated optimisation methodologies. Numerical models that predict the behaviour of these materials are developed with these methodologies. One of these methods is the asymptotic expansion homogenisation (AEH). This method is a powerful technique and has been widely used in modern engineering applications, such as, for example, nanotechnologies [1,2], smart composites modelling [3,4], and the modelling of thin network structures [5].

Applying the AEH, overall material properties can be derived from the mechanical behaviour of selected microscale representative volumes (representative unit-cells, RUC). Nevertheless, unit-cell based modelling requires the control of parameters such as reinforcement volume fractions, shapes and distributions within the matrix material. This leads to the development of automatic geometry generation algorithms. Additionally, unstructured tetrahedral meshes required by these types of RUC involve the control of specific periodic boundary conditions.

In the scope of this work, based on the mathematical methodology presented on part I of this work, a numerical simulation model is developed for three-dimensional finite element analyses (FEA) with asymptotic expansion homogenisation. Automatic representative unit-cell generation procedures are also developed, with control over relevant geometrical parameters. Additionally, specific algorithms are implemented for the association of degrees of freedom, in order to enforce periodicity boundary conditions over structured and unstructured finite element meshes. Numerical simulations are performed in order to validate the implemented procedures and to evaluate the influence of the non-periodicity of finite element meshes on the overall AEH results.

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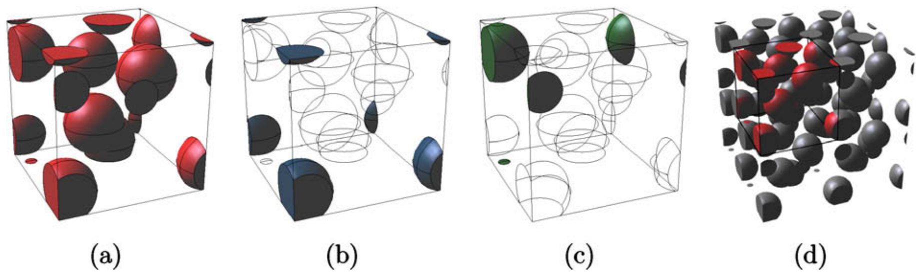

Fig. 1. Particle generation for periodical RUC: (a) random representative unit-cell, (b) corner particles, (c) face and edge particles and (d) unit-cell periodicity.

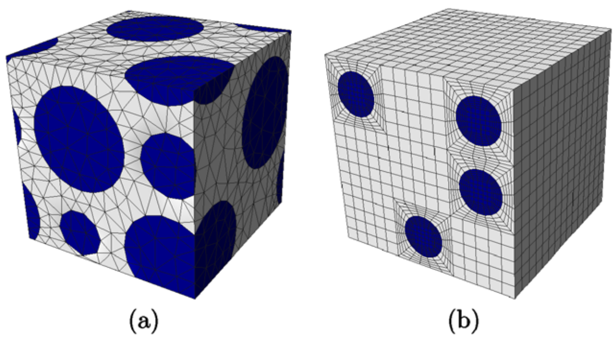

Fig. 2. Finite element meshes: (a) unstructured tetrahedral and (b) structured hexahedral meshes.

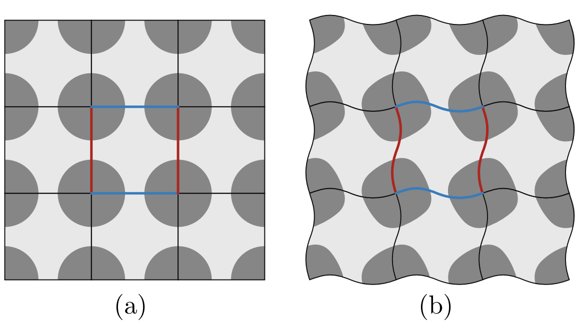

Fig. 3. RUC periodicity: (a) original and (b) deformed geometries.

2. Finite element method in AEH

2.1. Multiscale numerical equations – asymptotic expansion homogenisation

The asymptotic expansion homogenisation method is used to solve problems that involve physical phenomena on continuous media with a periodic microstructure. AEH is a useful technique to study the behaviour of structural components built with composite materials. For the elasticity problem, the AEH is an exact mathematical technique through which one can solve problems associated with differential partial operators with high-frequency periodic variations of its coefficients, solving a problem associated with a differential operator with constant coefficients [6,7]. This is called the homogenised elasticity problem. The coefficients of the homogenised problem are determined from the solution of a problem defined on the microscale unit-cell, enforcing periodic constraints to its boundaries [8]. One of the main advantages of this method lies on a significant reduction of the number of degrees of freedom of the elasticity problem. In fact, this technique allows the modelling of the microstructural details of the composite material based on a single representative unit-cell. The macroscale is modelled as an equivalent homogeneous body.

Another advantage of the asymptotic expansion homogenisation method is that it allows the characterisation of the microstructural strain and stress fields. Unlike other usual homogenisation methods, the AEH approach explicitly defines equations for this purpose. This process, designated by localisation, is essentially the inverse of the homogenisation.

As previously shown on part I of this work, the microscale problem is solved in two main steps. The first is the calculation of the corrector matrix ($\boldsymbol{\chi}$), which contains the eigendeformations of the representative periodic geometry [9]. Thus,

$$\int_{Y^e} \mathbf{B}^{\mathrm{T}}\mathbf{D}\mathbf{B}\,\mathrm{d}Y\boldsymbol{\chi} = \int_{Y^e} \mathbf{B}^{\mathrm{T}}\mathbf{D}\,\mathrm{d}Y = \mathbf{F}^{\mathrm{D}}, \qquad (1)$$

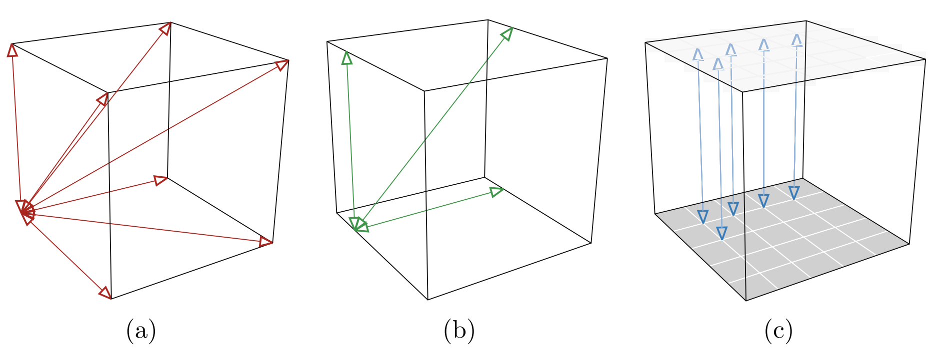

Fig. 4. Node association schemes in RUC boundaries: (a) in corners, (b) in edges and (c) in faces.