Available online at www.sciencedirect.com
ScienceDirect
Computational Materials Science 43 (2008) 337–344
COMPUTATIONAL MATERIALS SCIENCE
www.elsevier.com/locate/commatsci

An anti-plane shear crack in bonded functionally graded piezoelectric materials under electromechanical loading

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Received 2 September 2007; received in revised form 11 October 2007; accepted 28 November 2007
Available online 20 March 2008

Abstract

This paper is concerned with the problem of a Griffith crack in bonded functionally graded piezoelectric materials under the anti-plane shear loading. To make the analysis tractable, the properties of the functionally graded piezoelectric materials, such as elastic modulus, piezoelectric constant and dielectric constant, are assumed in exponential forms and vary along the crack direction. The crack surface condition is assumed to be electrically impermeable or permeable. Integral transform and dislocation density functions are employed to reduce the problem to the solution of a system of singular integral equations. The effects of the loading parameter λ, material constants and the geometry parameters on the stress intensity factor, the energy release ratio and the energy density factor are studied.

Keywords: Functionally graded piezoelectric materials; Impermeable; Permeable; Singular integral equation

1. Introduction

It is well known that piezoelectric materials produce an electric field when deformed and undergo deformation when subjected to an electric field. In piezoelectric materials and devices, the electrical and the mechanical loads are applied to the piezoelectric components which give rise to high stresses and can lead to their failure. A lengthy literature has now developed for the fracture mechanics of piezoelectric materials [1–6]. To improve the reliability and durability problems arising largely from high residual and thermal stress, poor interfacial bonding strength, functionally graded piezoelectric materials (FGPMs) as the new generation of composites have been developed. Chen et al. [7] considered the dynamic anti-plane problem for a functionally graded piezoelectric strip (FGPs) containing a central crack vertical to the boundary. Both the impermeable and permeable cases are considered. They applied integral transform and dislocation density functions to reduce the problem to solving Cauchy singular integral equations. Ueda [8] obtained the solutions for a crack in FGPs bonded to two elastic surface layers. He used the energy density factors to predict the fracture behavior of the structure. Chue and Ou [9] investigated mode III crack problems for two bonded functionally graded piezoelectric materials. The problem of a crack located in a functionally gradient piezoelectric interlayer is considered by Hu et al. [10]. Ma et al. [11] investigated the electro-elastic behavior of a Griffith crack in a functionally graded piezoelectric strip. Yong and Zhou [12] studied the anti-plane shear problem for a cracked functionally graded piezoelectric layer bonded to two piezoelectric half-planes.

In this present paper, the anti-plane shear problem for bonded functionally graded piezoelectric materials is investigated. The crack surface condition is assumed to be electrically impermeable or permeable. The standard Fourier transform technique is adopted to solve the governing equations for the functionally graded piezoelectric materials. By satisfying the boundary conditions and the interface

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0927-0256/$ - see front matter © 2008 Published by Elsevier B.V.
doi:10.1016/j.commatsci.2007.11.009

continuity conditions, the problem is reduced to two singular integral equations. The solution has been obtained numerically. The effects of the loading parameter λ, material constants and the geometry parameters on the stress intensity factor, the energy release ratio and the energy density factor are studied.

2. Formulation of the problem

Consider a crack in a FGPs of width h bonded to a dissimilar functionally graded piezoelectric material (FGPM). The crack is perpendicular to the interface and lies in $a < x < b$ as shown in Fig. 1. The crack length and the x-coordinate of the crack center are defined as $2a_0 = b - a$ and $c = (b + a)/2$, respectively.

All materials exhibit transversely isotropic behavior and are poled in the z-direction. An anti-plane shear loading and an electric displacement are applied on the crack surfaces. The constitutive equations can be written as

$$\begin{aligned}\tau_{xzk} &= c_{44k}(x)\frac{\partial w_k}{\partial x} + e_{15k}(x)\frac{\partial \varphi_k}{\partial x},\\ \tau_{yzk} &= c_{44k}(x)\frac{\partial w_k}{\partial y} + e_{15k}(x)\frac{\partial \varphi_k}{\partial y},\\ D_{xk} &= e_{15k}(x)\frac{\partial w_k}{\partial x} - \varepsilon_{11k}(x)\frac{\partial \varphi_k}{\partial x},\\ D_{yk} &= e_{15k}(x)\frac{\partial w_k}{\partial y} - \varepsilon_{11k}(x)\frac{\partial \varphi_k}{\partial y},\end{aligned} \tag{1}$$

where τ_{izk}, w_k, D_{ik} and ϕ_k ($i = x, y$, $k = 1, 2$) are the shear stresses, anti-plane displacements, in-plane electrical displacements and electric potentials, respectively, while subscripts $k = 1,2$ refer to the FGPs 1 and the FGPM 2. The variations of material constants $c_{44k}(x)$, $e_{15k}(x)$ and $\varepsilon_{11k}(x)$ called the shear modulus, piezoelectric constants, and dielectric constants, respectively, are assumed in the following exponential forms;

$$\begin{aligned}&c_{441}(x) = c_0 e^{\beta x}, \quad e_{151}(x) = e_0 e^{\beta x}, \quad \varepsilon_{111}(x) = \varepsilon_0 e^{\beta x},\\ &c_{442}(x) = c_0 e^{\gamma x}, \quad e_{152}(x) = e_0 e^{\gamma x}, \quad \varepsilon_{112}(x) = \varepsilon_0 e^{\gamma x},\end{aligned} \tag{2}$$

where β and γ are called nonhomogeneous parameters. The constants c_0, e_0, ε_0 are the material properties at $x = 0$.

The static equilibrium equation and Maxwell's equation under electro-static condition are given as

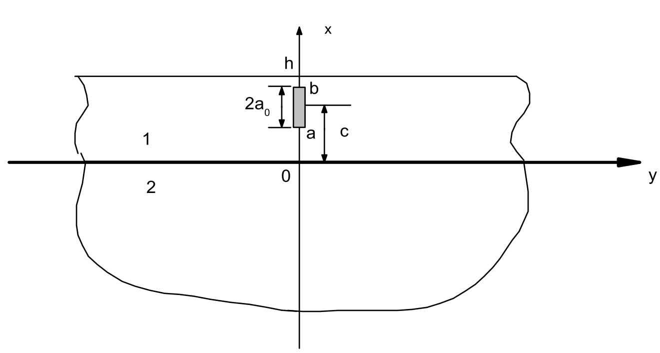

Fig. 1. Geometry of the crack problem.

$$\frac{\partial \tau_{xzk}}{\partial x} + \frac{\partial \tau_{yzk}}{\partial y} = 0, \quad \frac{\partial D_{xk}}{\partial x} + \frac{\partial D_{yk}}{\partial y} = 0, \quad k = 1, 2. \tag{3}$$

Substituting Eq. (1) into Eq. (3) and using the relation (2), we obtain the following equations:

$$\begin{cases} c_0\nabla^2 w_1 + e_1\nabla^2\varphi_1 + \beta\left(c_0\frac{\partial w_1}{\partial x} + e_1\frac{\partial \varphi_1}{\partial x}\right) = 0,\\ e_0\nabla^2 w_1 - \varepsilon_0\nabla^2\varphi_1 + \beta\left(e_0\frac{\partial w_1}{\partial x} - \varepsilon_0\frac{\partial \varphi_1}{\partial x}\right) = 0,\end{cases} \tag{4}$$

$$\begin{cases} c_0\nabla^2 w_2 + e_1\nabla^2\varphi_2 + \gamma\left(c_0\frac{\partial w_2}{\partial x} + e_1\frac{\partial \varphi_2}{\partial x}\right) = 0,\\ e_0\nabla^2 w_2 - \varepsilon_0\nabla^2\varphi_2 + \gamma\left(e_0\frac{\partial w_2}{\partial x} - \varepsilon_0\frac{\partial \varphi_2}{\partial x}\right) = 0,\end{cases} \tag{5}$$

where $\nabla^2 = \partial^2/\ \partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator.

By separating the homogeneous solution through an appropriate superposition, the problem may be reduced to a perturbation solution in which self-equilibration crack surface tractions are the only nonzero external loads. For the problem described in Fig. 1, the mixed boundary conditions are

$$\begin{aligned}&w_1(0,y) = w_2(0,y), \quad \varphi_1(0,y) = \varphi_2(0,y),\\ &\tau_{xz1}(0,y) = \tau_{xz2}(0,y), \quad D_{x1}(0,y) = D_{x2}(0,y),\\ &\tau_{xz1}(h,y) = 0, \quad D_{x1}(h,y) = 0,\\ &w_1(x,0) = 0, \quad \varphi_1(x,0) = 0, \ 0 \leqslant x < a, \ b < x \leqslant h,\\ &w_2(x,0) = 0, \quad \varphi_2(x,0) = 0, \ -\infty \leqslant x < 0,\\ &D_{y1}(x,0) = D(x), \quad \tau_{yz1}(x,0) = \tau(x), \ a < x < b\end{aligned} \tag{6}$$

for the impermeable case, and

$$\begin{aligned}&w_1(0,y) = w_2(0,y), \quad \varphi_1(0,y) = \varphi_2(0,y),\\ &\tau_{xz1}(0,y) = \tau_{xz2}(0,y), \quad D_{x1}(0,y) = D_{x2}(0,y),\\ &\tau_{xz1}(h,y) = 0, \quad D_{x1}(h,y) = 0,\\ &w_1(x,0) = 0, \quad 0 \leqslant x < a, \ b < x \leqslant h,\\ &\varphi_1(x,0) = 0, \quad 0 \leqslant x < h,\\ &w_2(x,0) = \varphi_2(x,0) = 0, \quad -\infty \leqslant x < 0,\\ &D_{y1}(x,0) = D_c(x,0) = D(x),\\ &\tau_{yz1}(x,0) = \tau(x), \quad a < x < b\end{aligned} \tag{7}$$

for the permeable case, where $D_c(x,0)$ denotes the electric displacement of the space of the crack itself.

3. Singular integral equations

Firstly, we proceed with the electrically impermeable case. By using Fourier transform method, we can obtain

$$\begin{cases} w_1(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A_1(\alpha)\exp(m_1 y)e^{-i\alpha x}d\alpha\\ \qquad + \frac{2}{\pi}\int_0^{\infty}[C_1(\alpha)\exp(n_1 x)\\ \qquad + [C_2(\alpha)\exp(n_2 x)\sin(\alpha y)d\alpha,\\ \varphi_1(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} B_1(\alpha)\exp(m_1 y)e^{-i\alpha x}d\alpha\\ \qquad + \frac{2}{\pi}\int_0^{\infty}[D_1(\alpha)\exp(n_1 x)\\ \qquad + [D_2(\alpha)\exp(n_2 x)\sin(\alpha y)d\alpha,\end{cases} \tag{8}$$

$$\begin{cases} w_2(x,y) = \frac{2}{\pi}\int_0^{\infty} E_2(\alpha)\exp(px)\sin(\alpha y)dx,\\ \varphi_2(x,y) = \frac{2}{\pi}\int_0^{\infty} F_2(\alpha)\exp(px)\sin(\alpha y)dx,\end{cases} \tag{9}$$