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A triphasic model of transversely isotropic biological tissue with applications to stress and biologically induced growth

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Abstract

A triphasic model (solid, interstices filled with water containing nutrients) is proposed for the phenomenological description of transversely isotropic saturated biological tissues including the phenomena of growth. This is done within the framework of a macro-mechanical description based on the Theory of Porous Media (TPM). Thereby, the constitutive equation of growth is determined by the state of stress and the local proportion of nutrients responsible for the mass exchange. The transversely isotropic behavior of the solid influences both the stress response and the internal fluid permeability, which is considered using an invariant formulation of the Helmholtz free energy and transversely isotropic permeability functions. After presenting the developed framework of the calculation concept including the representation of the governing weak formulations of the equations for large deformations, some representative numerical examples are examined.

Keywords: Biological tissue; Growth; Remodeling; Theory of Porous Media (TPM); Multiphase transitions; Conversion processes; Constitutive modelling

1. Introduction

Biological soft and hard tissues are highly complex materials capable of performing a wide range of functions. Two basic properties of biological tissues are optimized load transfer and the capacity for growth. The former is often realized by a combination of inner anisotropic structural components such as fibres situated in porous saturated tissue. Therefore, the optimized load transfer is solved for both the maximum stress and the hydrostatic pressure. The growth results from a phase transition inside the tissue where mass exchange between the biological skeleton and the nutrients, located in the saturating fluid, is observed. In many cases, e.g. healing of bone fracture, osteoporosis or wound healing in soft tissue, the growth is influenced by both stress and the chemical–biological situation of the tissue.

In the last 30 years, there has been an increasing interest in investigations into modeling the growth of biological tissues. This interest is a direct consequence of the fact that the body of acquired knowledge in the physics of biological growth processes and the wide range of applications associated with it has been increasing. With respect to comprehensive overviews on the experimental findings of the growth phenomena, the reader is referred e.g. to Fung [21,22] and Taber [25].

The first continuum theory for the description of growth in hard tissues was presented by Cowin and Hegedus [12]. Within the theory for non-polar one-component materials, the biological structure was considered as a so-called open system, i.e., supply terms of mass, momentum and energy were introduced. In Cowin and Hegedus [12] only volume sources were permitted. An extension of this model is presented in Nackenhorst [27] or Epstein and Maugin [31], see also Kuhl et al. [40] and Kuhl and Steinmann [41]. Apart from volume sources, they introduce additional surface

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fluxes of mass, momentum and energy. Another possibility for the modeling of growth is discussed, e.g. in Ambrosi and Mollica [33]. The basis of this description is a model of open systems which introduces a multiplicative decomposition of the deformation gradient into a growth part and a non-growth part, where only a volume source for mass creation and annihilation is considered. Furthermore, in Ambrosi and Mollica [33] a reaction–diffusion equation for nutrients is formulated, i.e., this model can be interpreted as a two phase model. Another approach to describe growth processes in bio-materials is the Theory of Porous Media (TPM) and the Mixture Theory (multiphase theories), see [39,44].

Currently, several groups are working in the field of porous media using either the Theory of Porous Media, the Mixture Theory or Biot's Theory, see e.g. the participants' contributions at the *IUTAM Symposium on Mechanics of Physicochemical and Electromechanical Interactions in Porous Media* (2003) or the *2nd Biot Conference on Poromechanics* (2002). The minor but significant differences between these approaches have been discussed in detail in e.g. [26,30] or [47] and therefore are not repeated in this paper. In these investigations, a description based on the Theory of Porous Media has been judged to be preferred. The Theory of Porous Media is the Mixture Theory, restricted by or combined with the concept of volume fractions, e.g. see [15,16,20,36,30] or [34]. With the Mixture Theory a multi-component continua can be described including internal interactions between the constituents. A broad review to the Mixture Theory can be taken, e.g. from [7,8,10,11,17]. In order to retain the possibility of identification for the individual constituents even in the mixture body, the Mixture Theory will be restricted by the concept of volume fractions. For more on this concept, the reader is referred to the work by e.g. [1,5,3,4,6].

Within the framework of the TPM and mixture theories, only volume sources of the individual constituents are considered, see e.g. [16,20,30]. The local supply terms of mass, momentum and energy are caused by the other constituents which simultaneously occupy the same position at the same time. Regarding the introduced surface fluxes for the individual constituents, it will be postulated that only one part of the volume source of the constituent is caused by the other phases. The remaining part is associated with a surface flux. This approach yields different structures of the balance equations compared to the models of one-component materials for open systems.

As aforementioned, the growth processes is driven by mechanical, chemical, genetic, metabolic and hormonal influences. Due to the lack of detailed knowledge and specific parameters to quantify all these influences, today a holistic numerical simulation cannot be provided. However, the capability of tissues (particularly hard tissues) of remodelling its structure and density due to a changing stress state has been well known for over a century. Moreover, the pre-condition for tissue growth is the existence of growth material like nutrients or similar. Therefore, in this work a calculation concept is presented for the description of stress and nutrient induced growth based on the Theory of Porous Media which leads to a highly coupled set of differential equations. The numerical solution has been treated within the framework of a standard Galerkin procedure whereby the resolving weak formulations are inserted into the finite element program FEAP. Thus, we are able to express the usefulness of the proposed theory by dint of numerical simulations.

However, first of all a consistent model has to be developed. This has been done based on the well established Theory of Porous Media for which basic relations will be summarized in the following section.

2. Theory of Porous Media

We consider a continuum which consists of several constituents. The investigated porous body consists of φ^{S} (solid) which is saturated by φ^{F} (fluid). The fluid itself is composed of φ^{L} (liquid) and φ^{N} (nutrients), see Fig. 1.

In the volume fraction concept it is assumed that the porous solid always models a control space and that the pores are statistically and homogeneously distributed. Therefore, the porous medium occupy the control space of the porous solid B_{S}, with the boundary ∂B_{S}, with real volumes v^{α}, where the index α denotes $\kappa \in \{S, L, N\}$ individual constituents. The boundary ∂B_{S} is a material surface for the solid phase and a non-material surface for the liquid and gas phases.

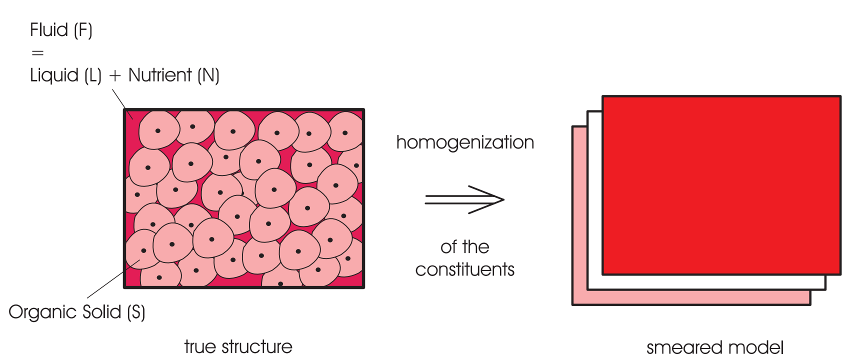

Fig. 1. Homogenization.