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On the effectiveness of porosity on unsteady flow between parallel plates of a viscoelastic fluid under constant pressure gradient with heat transfer

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Abstract

The unsteady flow in a porous medium of an incompressible non-Newtonian viscoelastic fluid between two parallel horizontal non-conducting porous plates is studied with heat transfer. A sudden uniform and constant pressure gradient and uniform suction and injection through the surface of the plates are applied. The two plates are kept at different but constant temperatures, while the Joule and viscous dissipations are taken into consideration. Numerical solutions for the governing momentum and energy equations are obtained using finite difference approximations. The effect of the porosity of the medium, the parameter describing the non-Newtonian behavior, and the velocity of suction and injection on both the velocity and temperature distributions is examined.

Keywords: Channel flow; Heat transfer; Non-Newtonian; Viscoelastic; Porous medium

1. Introduction

The flow of a viscous fluid between two parallel plates has applications in many devices, such as magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry, purification of molten metals from non-metallic inclusions and fluid droplets-sprays. The flow between parallel plates of a Newtonian fluid with heat transfer, subjected to different physical effects, have been studied by many authors [1–10]. These results are important for the design of the duct wall and the cooling arrangements. The rectangular channel problem has later been extended also to fluids obeying non-Newtonian constitutive equations. The flow of a viscoelastic fluid has attracted the attention of many authors [11–15] due to its important industrial applications [12].

In the present work, the flow between parallel plates in a porous medium of a non-Newtonian viscoelastic fluid is studied with heat transfer in the presence of a constant pressure gradient. The flow is subjected to a uniform suction from above and a uniform injection from below. The two plates are kept at two different but constant temperatures. The flow in the porous medium deals with the analysis in which the differential equation governing the fluid motion is based on Darcy's law which accounts for the drag exerted by the porous medium [16–18]. The viscous dissipation is taken into consideration in the energy equation. The governing momentum and energy equations are solved numerically using the finite difference approximations. The inclusion of the porosity effect, the non-Newtonian fluid characteristics as well as the velocity of suction and injection leads to some interesting effects, on both the velocity and temperature fields.

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2. Formulation of the problem

The fluid is assumed to be incompressible, viscoelastic and flows between two infinite horizontal parallel plates located at the $y = \pm h$ planes and extends from $x = -\infty$ to ∞ and from $z = -\infty$ to ∞. The upper and lower plates are kept at two constant temperatures T_2 and T_1, respectively, with $T_2 > T_1$. The flow is driven by a uniform and constant pressure gradient $\mathrm{d}P/\mathrm{d}x$ in the x-direction, and a uniform suction from the above and injection from below which are applied at $t = 0$. The flow is through a porous medium where the Darcy model is assumed [18]. The plates are assumed to be infinite in the x- and z-directions which makes the physical quantities to not change in these directions. Thus, the velocity vector of the fluid, in general, is given by:

$$\vec{v}(y,t) = u(y,t)\vec{i} + v_0\vec{j}$$

It is because of the conservation of mass, i.e., $\mathrm{div}\,\vec{v} = 0$ and due to the uniform suction the velocity component $v(y,t)$ is assumed to have a constant value v_0.

The fluid motion starts from rest at $t = 0$, i.e., $u = 0$ for $t \leqslant 0$. The no-slip condition at the plates implies that $u = 0$ at $y = \pm h$. It is also assumed that the initial temperature of the fluid is T_1, thus the initial and boundary conditions of temperature are $T = T_1$ at $t = 0$, $T = T_1$, at $y = -h$, $t > 0$ and $T = T_2$, at $y = h$, $t > 0$. The fluid motion is governed by the momentum equations [19]:

$$\rho\left(\frac{\partial u}{\partial t} + v_0\frac{\partial u}{\partial y}\right) = -\frac{\mathrm{d}P}{\mathrm{d}x} - \frac{\mu}{\bar{K}}u + \frac{\partial \tau_{xy}}{\partial y}, \tag{1}$$

where ρ is the density of the fluid, σ is the electric conductivity of the fluid, $\bar{K}$ is the Darcy permeability [16–18], the second term in the right-hand side represents the porosity force, and τ_{xy} is the component of the shear stress of the viscoelastic fluid given, respectively, as [10]:

$$\tau_{xy} = \mu\frac{\partial u}{\partial y} - \frac{\mu}{\alpha}\frac{\partial \tau_{xy}}{\partial t}, \tag{2}$$

where μ is the coefficient of viscosity and α is the modulus of rigidity. In the limit α tends to infinity or at steady state, the fluid behaves like a viscous fluid without elasticity. Solving Eq. (2) for τ_{xy} in terms of the velocity component u, we obtain

$$\frac{\partial \tau_{xy}}{\partial t} = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{1}{\alpha}\frac{\partial}{\partial y}\left(\mu\frac{\partial}{\partial t}\left(\mu\frac{\partial u}{\partial y}\right)\right), \tag{3}$$

where the term $(1/\alpha^2)\frac{\partial}{\partial y}\left(\mu\frac{\partial}{\partial t}\left(\mu\frac{\partial \tau_{xy}}{\partial t}\right)\right)$, which is proportional to $(1/\alpha^2)$, have been neglected. Substituting Eq. (3) in the momentum Eq. (1) yields

$$\rho\left(\frac{\partial u}{\partial t} + v_0\frac{\partial u}{\partial y}\right) = -\frac{\mathrm{d}P}{\mathrm{d}x} - \frac{\mu}{\bar{K}}u + \mu\frac{\partial^2 u}{\partial y^2} - \frac{\mu^2}{\alpha}\frac{\partial^3 u}{\partial t\partial y^2}. \tag{4}$$

The temperature distribution is governed by the energy equation [19]:

$$\rho c_p\left(\frac{\partial T}{\partial t} + v_0\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2, \tag{5}$$

where c_p and k are, respectively, the specific heat capacity at constant pressure and the thermal conductivity of the fluid. The second term on the right-hand side represents the viscous dissipation. Eqs. (4) and (5) can be made dimensionless by introducing the following dimensionless variables:

$$\hat{y} = \frac{y}{h}, \quad \hat{t} = \frac{\mu t}{h^2\rho}, \quad \hat{u} = \frac{\rho h u}{\mu}, \quad p = \frac{P\rho h^2}{\mu^2}, \quad \widehat{T} = \frac{T - T_1}{T_2 - T_1}.$$

We also define the following dimensionless parameters:

$S = \dfrac{\rho h v_0}{\mu}$, the suction parameter,

$M = h^2/\bar{K}$ is the porosity parameter,

$Pr = \dfrac{\mu c_p}{k}$, the Prandtl number,

$E_c = \dfrac{\mu^2}{\rho^2 h^2 c_p(T_2 - T_1)}$, the Eckert number,

$K = \dfrac{\mu^2}{\rho\alpha h^2}$, the viscoelastic parameter,

In terms of these dimensionless quantities, Eqs. (4) and (5) may be written, after dropping all hats for convenience, as:

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = -\frac{\mathrm{d}P}{\mathrm{d}x} - Mu + \frac{\partial^2 u}{\partial y^2} - K\frac{\partial^3 u}{\partial t\partial y^2}, \tag{6}$$

$$\frac{\partial T}{\partial t} + S\frac{\partial T}{\partial y} = \frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} + E_c\left(\frac{\partial u}{\partial y}\right)^2. \tag{7}$$

The initial and boundary conditions for the velocity and temperature in the dimensionless form are written as:

$$u(y,0) = 0, \quad u(-1,t) = u(1,t) = 0, \tag{8}$$

$$T(y,0) = 0, \quad T(-1,t) = 0, \quad T(1,t) = 1. \tag{9}$$

Eqs. (6) and (7) represent a system of partial differential equations which is solved numerically under the initial and boundary conditions (8) and (9), using the finite difference approximation. The Crank–Nicolson implicit method [17] is used at two successive time levels. Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximation in the y-direction. The diffusion terms are replaced by the average of the central differences at two successive time-levels. Finally, the resulting block tridiagonal system is solved using the generalized